Journal of Mathematics and Statistics

Original Research Paper

Stochastic Model for Pricing Normal Bonds when Maturity Periods Cross Over to Pandemic Period

[1]Sulaiman Sani and [2]Siphelele Lushaba

[1]*Department of Mathematics, University of Eswatini, Swaziland*
[2]*Sales and Audit, Eswatini Coca-Cola Beverages Company, Swaziland*

Article history
Received: 07-02-2023
Revised: 27-03-2023
Accepted: 14-04-2023

Corresponding Author:
Sulaiman Sani
Department of Mathematics,
University of Eswatini,
Swaziland
Email: ssulaiman@uniswa.sz

Abstract: In this study, Ito form for normal bonds trading where maturity periods cross over to COVID-19 pandemic period is presented. It is shown that normal bonds in this period experience path reversals respective to their canonical paths. The criterion used in arriving at this striking result is also presented. As a key recommendation, it is necessary that bondholders enact flexible pricing laws that strengthen the issuer to continue trading in the present COVID-19 pandemic time through the reverse path identified in this study.

Keywords: CAT Bond, Sub-Exponential Shock, Noise, Wiener Process, Issuer

Introduction

The advent of COVID-19 pandemic has altered the dynamics of many stochastic processes. As at the 6^{th} of February 2023, the number of infected persons stands at 671,706,853 to which 6,844,645 deaths were recorded[1]. The expectation that global bond traders are a significant fraction of these figures is not only likely but realistic. Liang (2020) asserts that if a significant chunk of bond traders lies in the COVID-19 infected class then the bond market incurs huge losses due to rapid sales at giveaway prices. Thus, if there are many COVID-19 infected bond traders in countries known for raising bonds, economic breakdown and meltdown may happen unless adequate trade precautions are taken. One precaution is the adoption of new hybrid bond pricing models that can curb emerging pandemic arbitrage in markets. For more on the subject of hybrid models for optimization purposes, we refer the reader to the works of (Alhawarat *et al*., 2021; Salleh *et al*., 2022).

Similarly, if many issuers die in a small time as in the case of the dreaded COVID-19 pandemic figures above, holders will struggle in receiving proportionate coupon payments. Under this condition, bond corporations will be unable to make capital payments on both secured and unsecured loans and will face serious challenges such as total collapse leading to loss of resources (Brown, 2006). The unfortunate collapse of silicon valley bank USA is a good example in this respect. Another realistic instance when bondholders will not receive payment from issuers is when bond-raising countries operate under lockdowns and other forms of COVID-19 restrictions. In this case, issuers are not fully paid salaries and wages due to changing work forms leading to a rising rate of defaults (Buchheit and Gulat, 2002). Here, transaction strains are developed and can lead to acute financial losses. To mitigate these losses and many others, newer bond pricing models for the COVID-19 pandemic time are needed to save investment resources.

For instance on 23^{rd} March 2020, the announcement for investment grade bonds was made. This announcement pushed the price of the bond by 7% on exercise with no signal effect on stock prices. This shocking bond-stock price path resulted in investors flocking to stocks instead of bonds (Haddad *et al*., 2021) because of additionally envisaged risks attached to bond dynamics sequel to the pandemic. At the moment, global bond market operates under additional risk that increases both rates and rise of defaults (Zaghini, 2023). Since COVID-19 pandemic has created additional risk measures in reality, then the need to incorporate this realism where maturity periods take longer than expected default times to wade away price informativeness (Dávila and Parlatore, 2018), or feedback effect (Sani *et al*., 2020; Edmans *et al*., 2015; Cofnas, 2016) and ripple effect (George and Beard, 2023) cannot be over-emphasized. In this case, the feedback effect will change speculators' decision on bonds which eventually lead to mispricing in both the short and long terms. For instance in the long term, bond markets will lose their fair share of profit margins to stocks since profit margins depend on the correctness of pricing models employed in both time and space (Joyner, 2006). Consequently, it is crucial to develop newer bond pricing models especially where maturity times cross over to the pandemic time. In this respect, we construct a new stochastic bond pricing model for cross-over bonds. First, the maturity time is

[1]Data retrieved from the John Hopkins University Dashboard on the 6th of February 2023@9.10 am

redefined in view of lengthy default tendencies (Goldie and Kluppelberg, 1988; Mikosch, 1999; Foss *et al*., 2013).

Chao and Zou (2018) studied flood related Catastrophe (CAT) bond prices and established the CIR-copula-POT model with stochastic rates showing that maturity dates affect bond prices. Shao (2015) considered both seismic and nuclear catastrophes and showed that bond prices are affected by extended maturities and defaults (Shao *et al*., 2015). Shao *et al*. (2017) studied CAT bond prices as a sequence of compound inhomogeneous Poisson processes disturbed by a diffusion process and came up with a semi-Markov model that examined the claim's inter-arrival times. Numerical analysis showed that the price of CAT bonds will decline when the threshold level is decreased, the time to maturity is increased and the likelihood of default also increased. Lee and Yu (2002) explored how CAT bonds relate to the financial market in the presence of rising default risks and showed that both moral hazards and baseline risks significantly can lower bond trade.

Nowak and Romaniuk (2017) studied CAT bonds related to investor risk-taking and showed that adjusted bond pricing models can wade away arbitrage tendencies better than non-adjusted bond pricing models (Romaniuk, 2003). Ma and Ma (2013) studied CAT bonds with non-homogeneous Poisson losses given interest rate uncert ainty, loss severity, and claim arrival intensities and showed that interest rates rise bond prices when the maturity time bracket is [0.25, 2.5]. Hofer *et al*. (2020) studied CAT bond pricing where associated risk is defined in a fixed interval and showed that threshold losses and expiration times have significant roles in arbitrage-free pricing. Tao *et al*. (2009) studied pricing models for earthquake-related catastrophes and stated the needed techniques for estimating personal insurance under seismic related deductibles. Vaugirard (2004) stated that the subject of pricing CAT bonds boils down to computing the first passage times for some jump diffusion stochastic processes under certain risk assumptions.

Egami and Young (2008) studied CAT bonds where inter-arrival claims follow the Poisson diffusion process and showed that prices are indifferent. Ma *et al*. (2017) developed a bond pricing model for zero coupon CAT bonds with stochastic Poisson arrivals and showed that hazards and interest rate risks affect zero coupon bond prices. In general, it is imperative to note that CAT bonds are hedging financial options as in Galeotti *et al*. (2013); Bodoff and Gan (2009); Lai *et al*. (2014); Gürtler *et al*. (2016). De Spiegeleer and Schoutens (2011); Baz and Chacko (2004); Rebonato (2018) described how existing bond models devoid of extra-uncertainties (Szczygielski *et al*., 2021; Lyócsa *et al*., 2020; Albulescu and Grecu 2023) are distinct from those ones where coupon payments assumed the paths of long-tailed processes (Norman *et al*., 2020). Thus for completeness, linking the times to maturity with the pandemic time is important for aided efficiency especially where default times assumed the path of the COVID-19 pandemic.

Materials and Methods

We consider a bond $\mathbb{B}$ whose non-pandemic trading time price at maturity $p(\mathbb{B}, T)$ is quoted as:

$$p(\mathbb{B},T)=\varepsilon\left(1-\left(\frac{1}{2}\right)^{(T)}\right)+\frac{\varphi}{\chi^{(T)}} \tag{1}$$

where, $T \geq 0$ is the time to maturity parameter of $\mathbb{B}$, ε is the ratio of monthly coupon payment to prevailing monthly market interest rate, χ is one on prevailing monthly market interest rate and ϕ is the face value of $\mathbb{B}$. We suppose that the same $\mathbb{B}$ is to be traded and exercised in the present COVID-19 pandemic period with $p(\mathbb{B}, T) \rightarrow p(\mathbb{B}, \bar{F}(T))$ such that:

$$p\left(\mathbb{B},\bar{F}(T)\right)=\varepsilon\left(1-\left(\frac{1}{\chi}\right)^{\bar{F}(T)}\right)+\frac{\varphi}{\chi^{\bar{F}(T)}} \tag{2}$$

Here, $\bar{F} \in \mathbb{S}$ is a bridge-in-price function sequel to changing trading intervals generated by the COVID-19 pandemic on $\mathbb{B}$ with the property that:

$$\lim_{T\to\infty}\frac{\bar{F}^{n*}(\mathbb{B},T)}{\bar{F}(\mathbb{B},T)}=n, n\geq 2 \tag{3}$$

If the path of $\bar{F}(\mathbb{B}, T)$ assumes the path of a wiener process $\{W(\mathbb{B}, T): T \geq 0\}$ through the maturity parameter $T \leq t$, then the guarantee of the existence of at least one natural homomorphism φ such that:

$$\phi:\mathbb{T}\times\Omega\rightarrow\mathbb{R} \tag{4}$$

is established. In this case, φ is a homomorphism between rings with identities onto $\mathbb{R}$.

Proposition 1

The homomorphism φ in (4) such that:

$$\phi:\mathbb{T}\times\Omega\times\bar{F}(T)\rightarrow\bar{F}\left(T,\Omega,W(T,\mathbb{B})\right) \tag{5}$$

is sub-exponential with parameter $\mathbb{N} \geq n$ provided that φ is measurable.

Proof

Fix $(\omega \in \Omega) \rightarrow x$ and apply the isomorphism of (Arnautov and Ermacova, 2014; Kulabokhov, 2019) on the right-hand side of (5). Consequently:

$$\lim_{T\to\infty}\frac{\bar{F}^{n*}\left(T,x,W\left(T,\mathbb{B}\right)\right)}{\bar{F}\left(T,x,W\left(T,\mathbb{B}\right)\right)}\gamma\lim_{T\to W(T,\mathbb{B})}\frac{\bar{F}^{n*}\left(W\left(T,\mathbb{B}\right)\right)}{\bar{F}\left(W\left(T,\mathbb{B}\right)\right)} \tag{6}$$

The mini parameter $\gamma \in \mathbb{R}$ is the intensity due to x fixed. Thus, one can write (6) in view of (3) respective of a natural number $\mathbb{N}$ as:

$$\lim_{T\to\infty}\frac{\bar{F}^{n*}\left(T,x,W\left(T,\mathbb{B}\right)\right)}{\bar{F}\left(T,x,W\left(T,\mathbb{B}\right)\right)}\leq\gamma\lim_{T\to W(T,\mathbb{B})}\frac{\bar{F}^{n*}\left(W\left(T,\mathbb{B}\right)\right)}{\bar{F}\left(W\left(T,\mathbb{B}\right)\right)}\leq\mathbb{N} \tag{7}$$

Since, $\mathbb{N} \geq 2$ in (7), the bond price $p(\mathbb{B}, T)$ in (1) must admit some sub-exponentially due to induced COVID-19 pandemic effects as in (2). In this case, the quoted price $p(\mathbb{B}, \bar{F}(T))$ in (2) holds good whenever $\mathbb{B}$ is within the dreaded[2] pandemic period with the representation[3] as in (7) and such that:

$$p\left(\mathbb{B},\bar{F}(T)\right)=\varepsilon\left(1-\left(\frac{1}{\chi}\right)^{\mathbb{N}}\right)+\frac{\varphi}{\chi^{\mathbb{N}}} \tag{8}$$

Equation (8) is an Ito process (Øksendal and Øksendal, 2003). Let:

$$p\left(\mathbb{B},\bar{F}\left(T,\mathbb{B}\right)\right)=p_T=g\left(T,x,W\left(T,\mathbb{B}\right)\right) \tag{9}$$

such that:

$$g\left(\bar{F}\left(T,x,W\left(T,\mathbb{B}\right)\right)\right)\in\mathbb{C}^2\left(\left[0,\infty\right)\right)\times\mathbb{R} \tag{10}$$

Then by Ito's formula on g(.), one obtains that:

$$\frac{\partial p\left(\mathbb{B},\bar{F}(T)\right)}{\partial T}=-\ln\varepsilon\frac{\partial\bar{F}\left(T,W\left(T,\mathbb{B}\right)\right)}{dT}p\left(\mathbb{B},\bar{F}(T)\right) \tag{11}$$

$$C_0=-\ln\varepsilon\frac{d\bar{F}\left(T,W\left(T,\mathbb{B}\right)\right)}{dT} \tag{12}$$

and that:

$$\frac{\partial p\left(\mathbb{B},\bar{F}(T)\right)}{\partial W\left(T,\mathbb{B}\right)}=-\ln\varepsilon\frac{\partial\bar{F}\left(T,W\left(T,\mathbb{B}\right)\right)}{\partial W\left(T,\mathbb{B}\right)}p\left(\mathbb{B},\bar{F}(T)\right) \tag{13}$$

Here:

$$C_1-\ln\varepsilon\frac{\partial\bar{F}\left(T,W\left(T,\mathbb{B}\right)\right)}{\partial W\left(T,\mathbb{B}\right)} \tag{14}$$

Finally:

$$\frac{\partial^2 p\left(\mathbb{B},\underline{F}(T)\right)}{\partial W^2(T)}=\left(C_2+C_3\right)p\left(\mathbb{B},\bar{F}(T)\right) \tag{15}$$

Here again:

$$C_2-\ln\varepsilon^2\frac{\partial^2\bar{F}\left(T,W\left(T,\mathbb{B}\right)\right)}{\partial W^2\left(T,\mathbb{B}\right)} \tag{16}$$

And that:

$$C_3-\ln\varepsilon^2\frac{\partial^2\bar{F}\left(T,W\left(T,\mathbb{B}\right)\right)}{\partial W^2\left(T,\mathbb{B}\right)} \tag{17}$$

Hence, by combining (11), (13), and (15), one obtains[4] that:

$$dp_T=\left(C_0+\frac{1}{2}\left(C_2+C_3\right)\right)p_T dT+C_1\,p_T dW\left(T,\mathbb{B}\right) \tag{18}$$

Corollary 1

Under (3) and (4), the price of $\mathbb{B}$ fluctuates with drift α and volatility σ given by:

$$\alpha=C_0+\frac{1}{2}\left(C_2+C_3\right) \tag{19}$$

$$\sigma=C_1 \tag{20}$$

Proof

This result is a consequence of (18) as in (Øksendal and Øksendal, 2003). Thus, if the COVID-19 process hits the bond price process in (1), then the result is a transformed process (8) whose features are those of the geometric Brownian motion (18). The path of (18) $(T\times\omega\times W(T,\mathbb{B}))$ $\to (p_\omega(T, W(T, \mathbb{B})))$ is measurable in view of (Øksendal and Øksendal, 2003). Solving for p_T gives:

$$p_T=p_{(0)}\exp\left(\left(\alpha-\frac{1}{2}\sigma^2\right)T+\sigma W\left(T,\mathbb{B}\right)\right) \tag{21}$$

[2] $\varepsilon=\frac{C_m}{r_m}$

[3] $\chi = 1 + r_m$

[4] $\alpha=\left[C_0+\frac{1}{2}\left(C_2+C_3\right)\right]$

Table 1: α, σ Trends versus T in COVID-19 Times

T(years)	α	σ
0	11.51300000	0.0000000
1	21.34200000	3.4112000
2	5.92190000	0.8528100
3	1.62960000	0.2076000
4	0.64778000	0.0631710
5	0.33685000	0.0233970
10	0.05464200	0.0008679
35	0.00181910	1.7451e-06
50	0.00065513	2.9402e-07
70	0.00024677	5.4734e-08
85	0.00013987	2.0741e-08
100	8.67930e-05	9.2048e-09

Table 1 shows α and σ paths for selected T. From the said table, the impact of the noise parameters can be deduced. For instance, in 2023, bond prices will drift about 65% of their non-pandemic face values at 6% frequency due to the COVID-19 pandemic. Interestingly in 2029, the price of the same asset drifts only around 5% at a frequency lesser than 1%.

Results and Discussion

For further numerical discussions, we simulate (21) to study the path of (2) in the light of the 80-20 Pareto distribution assuming that 80% of bond wealth is in the hands of 20% bondholders who are infected by the COVID-19 pandemic. We also simulate the bond price model in (1) to study the changes in the path between the two models. The following remarks hold good.

Remark 1

If T is Pareto 80-20 and positively increasing in R sequel to defaults, then the constants C_0, C_1, and C_3 decrease while C_2 increases.

Table 2 presents the constant values C_0, C_1, C_2 and C_3 against $T \leq 100$ years. From the said table, it is clear that C_0 values decrease slowly to 0.05 at $T = 10$ years. This trend is followed by a constant pattern of $C_0 \simeq 0$ for $T>10$ years. Clearly, the general trend for C_0 is that of a bond constant decreasing as T increases. In this case, C_0 represents the component of the prevailing interest rate during the pandemic. From Table 2 again, C_1 decreases to 0.02 where $T = 5$ years. This trend is followed by $C_1 \simeq 0$ afterward for all $T>5$ years. As a trend, C_1 decreases as T increases in the pandemic period. The same trend in path is taken by C_3 as a function of T. On the contrary, C_2 starts with a value of 3.4112 at $T = 1$ year and steadily increases to 0.04 at time $T = 4$ years. It can be conjectured that the decreasing $C_i{'}_s$ emanated from the hidden effects of the COVID-19 pandemic on the existing model (1).

Remark 2

Suppose B is quoted where T is sub-exponentially 80-20 Pareto-tailed sequel to the COVID-19 pandemic or any of its arguments. Then $p(\mathbb{B}, \bar{F}(\text{T}))$ is astronomical in the short term.

Table 3 presents the path analysis of $p(\mathbb{B}, \bar{F}(T))$ as in (18) and that of p as in (1). Clearly, at $T = 1$ year, p_T skyrockets from \$1000 to an astronomical value of \$16,729,000,000. Again, At $T = 2$ years and $T = 3$ years respectively, the bond price values are \$370,310,000 and \$232,080. Here, the COVID-19 pandemic created 80% uncertainties in the bond market causing investors to flee to other financial markets as in Ma and Ma (2013) under high-level uncertainty, trader loss severity, and high claim arrival intensities. Bond traders will choose to cash in to treat COVID-19 pandemic symptoms without minding the price. As a result, bond markets lose control over the short-term maturities. Afterward, bond prices show signs of stability in both time and space. For instance, in the year 2029, the bond exercised at \$1000 will trade at \$1494 via the canonical model and at \$1742 via the stochastic model developed in this study.

Remark 3

Under the 80-20 pareto COVID-19 pandemic tail, the price of $\mathbb{B}$ reverses its path to a monotonically decreasing path within the same maturity interval.

This information is clear from Table 3 showing that the price of the bond p climbs consistently from \$1000 at $T = 0$ to approximately \$6000 at $T = 100$ years. On the other hand, $p(\mathbb{B}, \bar{F}(T))$ declines towards extremely low values close to \$1000 as time to maturity T approaches 100 years. In this case, the paths assumed by the two models are opposite in direction and strictly monotonic agreeing with (Shao *et al*., 2015) that showed that the price of CAT bonds is most likely to reverse when the threshold level is decreased, the time to maturity is increased and the likelihood of default increases. This further proves the strength of the methodology designed and presented in this study.

Table 2: C_0, C_1, C_2, C_3 versus T in COVID-19 times

T(years)	C_0	C_1	C_2	C_3
0	11.5130000	0.0000000	0.0000000	0.0000000
1	3.41120000	3.4112000	-3.4112000	39.273000
2	1.43910000	0.8528100	-0.8528100	9.8183000
3	0.73683000	0.2076000	-0.1698500	1.9555000
4	0.42640000	0.0631710	-0.0421140	0.4848600
5	0.26852000	0.0233970	-0.0129980	0.1496500
10	0.05330100	8.6791e-04	-2.5530e-04	2.9389e-03
35	0.00181830	1.7451e-06	-1.4933e-07	1.7193e-06
50	0.00065504	2.9402e-07	-1.7627e-08	2.0294e-07
70	0.00024676	5.4734e-08	-2.3448e-09	2.6995e-08
85	0.00013987	2.0741e-08	-7.3182e-10	8.4254e-09
100	8.67910e-05	9.2048e-09	-2.7609e-10	3.1786e-09

Table 3: p and p_T versus T in COVID-19 times

T(years)	P($)	P_T($)
0	1000.0	1000.0000000
1	1049.5	1.6729e11
2	1098.9	3.7031e08
3	1148.4	2.3208e05
4	1197.8	17044.0000000
5	1247.1	6048.8000000
10	1493.6	1742.1000000
35	2717.0	1065.8000000
50	3443.7	1033.3000000
70	4404.2	1017.4000000
85	5118.3	1012.0000000
100	5827.1	1008.7000000

Table 4: p and p_T versus T in COVID-19 times

T(years)	P($)	P_T($)
0.0	1000.0	1000.0000000
1.0	1049.5	1.6729e11
2.0	1098.9	3.7031e08
3.0	1148.4	2.3208e05
4.0	1197.8	17044.0000000
5.0	1247.1	6048.80000000
10.0	1493.6	1742.1000000
11.0	1542.9	1608.9000000
11.1	1547.8	1597.8000000
11.2	1552.7	1587.0000000
11.3	1557.6	1576.5000000
11.4	1562.6	1566.3000000
11.5	1567.5	1556.3000000
11.6	1572.4	1546.7000000
11.7	1577.3	1537.3000000
11.8	1582.2	1528.2000000
11.9	1587.2	1519.3000000
12	1592.1	1510.7000000
13	1641.3	1435.6000000
35	2717.0	1065.8000000

Remark 4

There exists a unique price for B identical to both models. Table 4 shows $p(\mathbb{B}, \bar{F}(T))$ when $T = 11.2$ years is \$1587 which coincides with p at $T = 11.9$ years. At this point, the coincidental price represents the equilibrium price where the canonical model perfectly transformed itself onto the stochastic model. By proposition 1, $p(\mathbb{B}, \bar{F}(T))$ extends the canonical model onto the COVID-19 pandemic times at this maturity period. As a consequence, it is clear that the stochastic model has a root in the canonical model as claimed and analyzed in this study.

Remark 5

The 80-20 pareto tail containing COVID-19 trading times are normal bonds under long T.

Table 3 shows $p(\mathbb{B}, \bar{F}\ T))$ against selected values of T. From the said table, it is clear that a bond with a \$1000 face value has $p(\mathbb{B}, \bar{F}(T))$ ranging from \$1742.1 to \$1008.7 for $T \geq 10$. Clearly here, the pattern of long maturity times is being much closer to the face value of the bond during the COVID-19 pandemic. Again, bond prices enter the normal region in volatility hence, normal under maturity periods $T \geq 10$.

Conclusion

We design a CAT bond pricing model for normal bond trading during the COVID-19 pandemic time and provide its Ito representation and analysis. The work discovered a major result showing the existence of a relationship between CAT bonds of the COVID-19 pandemic with that of earthquakes as in Romaniuk (2003). Again, we have shown the existence of path reversals for COVID-19 pandemic CAT bonds. This result agrees with (Shao *et al*., 2015) for earthquakes CAT bonds. As a recommendation, since CAT bonds with short-term maturity periods are astronomically expensive in the 80-20 pareto tail region containing the COVID-19 pandemic, it is better to issue bonds in the mid-term to wave foreseen uncertainties. Finally, the work posits that CAT bonds within 80-20 pareto tail containing COVID-19 trading times attained steadiness under long maturity periods with equilibrium points around the eleventh year of the COVID-19 pandemic.

Acknowledgment

We are grateful to all the authors of the literature sources used in this article and the reviewers.

Funding Information

This article is funded by the Journal of Mathematics and Statistics (JMSS) completely. We are immensely grateful to the management and the editorial board of the JMSS for the invitation, sponsoring and funding the entire publication process of this article.

Competing Interests and Declarations

Consent to Participate

All authors have given their consent to participate in the entire process of this article.

Consent for Publication

All authors have consented to publish this article.

Author's Contributions

Sulaiman Sani: Pens down the article and proves the remarks together with the methods.

Siphelele Lushaba: Provides the literature backing of the article and the sample path analysis leading to the tables and figures studied in this study.

Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.

References

Albulescu, C. T., & Grecu, E. (2023). Government interventions and sovereign bond market volatility during COVID-19: A Quantile analysis. *Mathematics*, *11*(5), 1171. https://doi.org/10.3390/math11051171

Alhawarat, A., Salleh, Z., & Masmali, I. A. (2021). Convex combination between two different search directions of conjugate gradient method and application in image restoration. *Mathematical Problems in Engineering*, *2021*, 1-15. https://doi.org/10.1155/2021/9941757

Arnautov, V., & Ermacova, G. (2014). On the number of group topologies on countable groups. *Buletinul Academiei de Ştiinţe a Moldovei. Matematica*, *74*(1), 101-112. https://ibn.idsi.md/vizualizare_articol/31445

Baz, J., & Chacko, G. (2004). *Financial derivatives: Pricing, applications, and mathematics*. Cambridge University Press. ISBN-10: 9780521815109.

Bodoff, N. M., & Gan, Y. (2009, May). An analysis of the market price of cat bonds. In *CAS E-Forum*. https://www.casact.org/sites/default/files/database/forum_09spforum_completesp09.pdf#page=43

Brown, P. J. (2006). *An introduction to the bond markets*. John Wiley & Sons. ISBN-10: 9780470030745.

Buchheit, L. C., & Gulati, G. M. (2002). Sovereign bonds and the collective will. *Emory LJ*, *51*, 1317. https://heinonline.org/HOL/LandingPage?handle=hein.journals/emlj51&div=38&id=&page=

Chao, W., & Zou, H. (2018). Multiple-event catastrophe bond pricing based on CIR-Copula-POT model. *Discrete Dynamics in Nature and Society*. https://doi.org/10.1155/2018/5068480

Cofnas, A. (2016). *Trading Binary Options: Strategies and Tactics*. John Wiley & Sons. ISBN-10: 9781119194170.

Dávila, E., & Parlatore, C. (2018). *Identifying price informativeness* (No. w25210). National Bureau of Economic Research. https://www.nber.org/papers/w25210

De Spiegeleer, J., & Schoutens, W. (2011). *The handbook of convertible bonds: Pricing, strategies and risk management*. John Wiley & Sons. ISBN-10: 9781119978060.

Edmans, A., Goldstein, I., & Jiang, W. (2015). Feedback effects, asymmetric trading and the limits to arbitrage. *American Economic Review*, *105*(12), 3766-3797. https://www.aeaweb.org/articles?id=10.1257/aer.20141271

Egami, M., & Young, V. R. (2008). Indifference prices of structured catastrophe (CAT) bonds. *Insurance: Mathematics and Economics*, *42*(2), 771-778. https://doi.org/10.1016/j.insmatheco.2007.08.004

Foss, S., Korshunov, D., Zachary, S., Foss, S., Korshunov, D., & Zachary, S. (2013). Subexponential distributions. *An Introduction to Heavy-Tailed and Subexponential Distributions*, 43-74. https://doi.org/10.1007/978-1-4614-7101-1_3

Galeotti, M., Gürtler, M., & Winkelvos, C. (2013). Accuracy of premium calculation models for CAT bonds-an empirical analysis. *Journal of Risk and Insurance*, *80*(2), 401-421. https://doi.org/10.1111/j.1539-6975.2012.01482.x

George, P., & Beard, S. J. (2023). Ripple effect: How Silicon Valley Bank collapse is affecting other US banks. https://www.usatoday.com/story/graphics/2023/03/13/silicon-valley-bank-ripple-effect-banks/11463159002/.

Goldie, C. M., & Klüppelberg, C. (1998). Subexponential distributions. *A Practical Guide to Heavy tails: Statistical Techniques and Applications*, 435-459. ISBN-10: 9780817639518.

Gürtler, M., Hibbeln, M., & Winkelvos, C. (2016). The impact of the financial crisis and natural catastrophes on CAT bonds. *Journal of Risk and Insurance*, *83*(3), 579-612. https://doi.org/10.1111/jori.12057

Haddad, V., Moreira, A., & Muir, T. (2021). When selling becomes viral: Disruptions in debt markets in the COVID-19 crisis and the Fed's response. *The Review of Financial Studies*, *34*(11), 5309-5351. https://doi.org/10.1093/rfs/hhaa145

Hofer, L., Zanini, M. A., & Gardoni, P. (2020). Risk-based catastrophe bond design for a spatially distributed portfolio. *Structural Safety*, *83*, 101908. https://doi.org/10.1016/j.strusafe.2019.101908

Joyner, M. (2006). *The Great Formula: For Creating Maximum Profit with Minimal Effort*. John Wiley & Sons. ISBN-10: 9780470038888.

Kulabokhov, V. S. (2019). General Principle of Isomorph ism: Determining Inverses. *Symmetry*, *11*(10), 1301. https://doi.org/10.3390/sym11101301

Lai, V., Parcollet, M., & Lamond, B. F. (2014). The valuation of catastrophe bonds with exposure to currency exchange risk. *International Review of Financial Analysis*, 33, 243-252.

Lee, J. P., & Yu, M. T. (2002). Pricing default-risky CAT bonds with moral hazard and basis risk. *Journal of Risk and Insurance*, 25-44. https://www.jstor.org/stable/1558638

Liang, N. (2020). Corporate Bond Market Dysfunction During COVID-19 and Lessons from the Fed's Responses. *Available at SSRN 3725503*. http://dx.doi.org/10.2139/ssrn.3725503

Lyócsa, Š., Baumöhl, E., Výrost, T., & Molnár, P. (2020). Fear of the coronavirus and the stock markets. *Finance Research Letters*, *36*, 101735. https://doi.org/10.1016/j.frl.2020.101735

Ma, Z. G., & Ma, C. Q. (2013). Pricing catastrophe risk bonds: A mixed approximation method. *Insurance: Mathematics and Economics*, *52*(2), 243-254. https://doi.org/10.1016/j.insmatheco.2012.12.007

Ma, Z., Ma, C., & Xiao, S. (2017). Pricing zero-coupon catastrophe bonds using EVT with doubly stochastic Poisson arrivals. *Discrete Dynamics in Nature and Society*, *2017*. https://doi.org/10.1155/2017/3279647

Mikosch, T. (1999). Regular variation, Sub-exponentiality and their applications in probability theory. (Report Eurandom; Vol. *99013*). Eurandom.

Norman, J., Bar-Yam, Y., & Taleb, N. N. (2020). Systemic risk of pandemic via novel pathogens-Coronavirus: A note. *New England Complex Systems Institute*, *26*, 1-2.

Nowak, P., & Romaniuk, M. (2017). Catastrophe bond pricing for the two-factor Vasicek interest rate model with automatized fuzzy decision making. *Soft Computing*, *21*, 2575-2597. https://doi.org/10.1007/s00500-015-1957-1

Øksendal, B., & Øksendal, B. (2003). *Stochastic differential equations* (pp. 65-84). Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-14394-6_5

Rebonato, R. (2018). *Bond pricing and yield curve modeling: A structural approach*. Cambridge University Press. ISBN-10: 9781107165854.

Romaniuk, M. (2003). Pricing the risk-transfer financial instruments via Monte Carlo methods. *Systems Analysis Modelling Simulation*, *43*(8), 1043-1064. https://doi.org/10.1080/0232929031000150382

Salleh, Z., Almarashi, A., & Alhawarat, A. (2022). Two efficient modifications of AZPRP conjugate gradient method with sufficient descent property. *Journal of Inequalities and Applications*, *2022*, 1-21. https://doi.org/10.1186/s13660-021-02746-0

Sani, S., Maseko, S. P., Dlamini, Q., & Abdullahi, F. A. (2020). Pricing Pseudo Contingencies on Motion Picture Assets under No Free Lunch with Vanishing Risk. *Journal of Mathematical Finance*, *10*(04), 525. https://doi.org/10.4236/jmf.2020.104032

Shao, J. (2015). *Modelling catastrophe risk bonds* (Doctoral dissertation, University of Liverpool). https://core.ac.uk/download/pdf/80774192.pdf

Shao, J., Pantelous, A., & Papaioannou, A. D. (2015). Catastrophe risk bonds with applications to earthquakes. *European Actuarial Journal*, *5*(1), 113-138. https://doi.org/10.1007/s13385-015-0104-9

Shao, J., Papaioannou, A. D., & Pantelous, A. A. (2017). Pricing and simulating catastrophe risk bonds in a Markov dependent environment. *Applied Mathematics and Computation*, *309*, 68-84. https://doi.org/10.1016/j.amc.2017.03.041

Szczygielski, J. J., Bwanya, P. R., Charteris, A., & Brzeszczyński, J. (2021). The only certainty is uncertainty: An analysis of the impact of COVID-19 uncertainty on regional stock markets. *Finance Research Letters*, *43*, 101945. https://doi.org/10.1016/j.frl.2021.101945

Tao, Z., Tao, X., & Li, P. (2009, July). Pricing model for earthquake CAT bonds. In *2009 International Conference on Business Intelligence and Financial Engineering* (pp. 740-744). IEEE. https://ieeexplore.ieee.org/abstract/document/5208747

Vaugirard, V. (2004). A canonical first passage time model to pricing nature-linked bonds. *Economics Bulletin*, *7*(2), 1-7.

Zaghini, A. (2023). The Covid pandemic in the market: Infected, immune and cured bonds. *Journal of Financial Services Research*, 1-22. https://doi.org/10.1007/s10693-022-00394-z