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An Optimal Inventory Policy for Items having Linear Demand and Variable Deterioration Rate with Trade Credit

Singh Sarbjit and Singh Shivraj
Institute of Management Technology, Nagpur, Devnagri (PG) College, Meerut

Abstract: Problem statement: Demand considered in most of the classical inventory models is constant, while in most of the practical cases the demand changes with time. In this study model has been framed to study the items whose demand changes with time and deterioration rate increases with time. The effect of permissible delay is also incorporated in this study. The objective of this research is to develop an inventory model for perishable items whose perish-ability rate as well as demand increases with time **Approach:** Firstly, problem is framed in the form of linear differential equation model and this model had been solved using general solution techniques of linear differential equations. The solution obtained gives the inventory level at any particular time of the cycle period. With the help of this inventory level, total as well as average inventory cost has been obtained. **Results:** This study developed a model to determine an optimal order quantity by using calculus technique of maxima and minima. Thus it helps retailer to decide its optimal ordering quantity under the constraints of variable deterioration rate and linear pattern of demand. **Conclusion:** Numerical solution of the suggested model had also been proposed, the above model can be converted into constant demand model, or for items having no deterioration. This study can further be extended for items having some other demand pattern, also time value of money and inflation can be incorporated in this model to make it more realistic and present business environment suited.

Key words: Credit period, linear demand, variable deterioration rate, optimal payment time

INTRODUCTION

The concept of permissible delay is not new, it has been there from time immortal, even when currency was not in circulation and barter system was prevalent, than also permissible delay was provided by suppliers to buyers. In general practice, suppliers are known to offer their customers a fixed period of time and do not charge any interest for this period. However, a higher interest is charged if the payment is not settled by the end of credit period. The permissible delay in payment produces three advantages to the supplier Firstly it helps to attract new customer as it can be considered some sort of loan. Secondly it helps in the bulk sale of goods. The existence of credit period serves to reduce the cost of holding stock to the user, because it reduces the amount of capital invested in stock for the duration of the credit period. Thirdly it may be applied as an alternative to price discount because it does not provoke competitors to reduce their prices and thus introduce lasting price reductions.

Goyal[7] was the first to introduce the concept of permissible delay. Aggarwal and Jaggi[1] then extended Goyal's model for deteriorating items. Jamal *et al*.[10] further generalized the model to allow for shortages and deterioration. Teng[11] considered the EOQ under condition of permissible delay in payment which is further extended by Ken Chung Kun and Yun-Fu-Huang[4] for limited storage capacity. Goyal[12] developed the optimal pricing and ordering policies for items under permissible delay. Ghare and Schrader[6] were the first to use the concept of deterioration followed by Covert and Philip[5] who formulated a model with variable rate of deterioration with two parameter Weibull distributions, which was further extended by Shah[8]. Buzacott[3], Bierman and Thomas[2] investigated the inventory models with inflation followed by Misra[11]. Mishra also developed a discount model in which the effect of both inflation and time value of money was considered. Chandra and Bahner[4] had also developed model under inflation and time value of money. Chang[3] presented model for the situation where the demand rate is a time-continuous function and items deteriorate at a constant rate with partial backlogging. Su *et al*.[14] also developed model under inflation for stock dependent consumption rate and exponential decay. Sarbjit and Shivraj[10] developed an inventory model having linear demand rate under

Corresponding Author: Singh Sarbjit, Institute of Management Technology, Nagpur, Devnagri (PG) College, Meerut

permissible delay in payments with constant rate of deterioration.

This study is related to products having linear demand and variable rate of deterioration, which is the extension of author's earlier paper having linear demand pattern with constant rate of deterioration. Generally in trade credit, if cycle period is more than permissible delay, the buyer settles its account only after completion of cycle period. Plus most of the models on permissible delay consider cost price and selling price equal which is not feasible. But it is a fact that it is more beneficial to settle the account in between the permissible delay and cycle period. This time is known as payment time. With the help of this model retailer can easily extract its cycle period as well as payment time to reduce the total inventory cost.

MATERIALS AND METHODS

Mathematical model and analysis: In this model variable rate of demand is considered with variable rate of deterioration. Depletion of the inventory occurs due to demand (supply) as well as due to deterioration which occurs only when there is inventory i.e., during the period [0, T]. For this period the inventory at any time t is given by:

$$\frac{dI(t)}{dt}+\theta tI(t)=-(a+bt) \quad 0\le t\le T \tag{1}$$

$$I(t)e^{\vartheta\frac{t^2}{2}}=-a\left(t+\frac{\vartheta t^3}{6}\right)-b\left(\frac{t^2}{2}+\frac{\vartheta t^4}{8}\right)+C$$

where, at t = 0, I (t) = I_0 (initial inventory level). Putting this value in the above equation, we get:

$$I_o=C$$

This gives:

$$I(t)=I_oe^{-\vartheta\frac{t^2}{2}}-a\left(t+\frac{\vartheta t^3}{6}\right)e^{-\vartheta\frac{t^2}{2}}-b\left(\frac{t^2}{2}+\frac{\vartheta t^4}{8}\right)e^{-\vartheta\frac{t^2}{2}} \quad 0\le t\le T \tag{2}$$

It is obvious that at t = T i.e., at the end of cycle period, I (T) = 0. So Eq. 2 yields:

$$I_o=a\left(T+\frac{\vartheta T^3}{6}\right)+b\left(\frac{T^2}{2}+\frac{\vartheta T^4}{8}\right) \tag{3}$$

Substituting the value of I_o in Eq. 2 we get:

$$I(t)=\begin{bmatrix} a\left\{(T-t)+\frac{\vartheta}{6}(T^3-t^3)\right\} \\ +b\left\{\left(\frac{T^2}{2}-\frac{t^2}{2}\right)+\frac{\vartheta}{8}(T^4-t^4)\right\}\end{bmatrix} e^{-\vartheta\frac{t^2}{2}} \quad 0\le t\le T \tag{4}$$

The total demand during cycle period T is $\int_0^T (a+bt)dt$. Thus it can be easily seen that the amount of items deteriorates during one cycle is given by:

$$D_T=I_o-\int_0^T (a+bt)dt=\frac{a\vartheta T^3}{6}+\frac{b\vartheta T^4}{8} \tag{5}$$

Evaluating the cost functions:

- For most of the inventory systems ordering cost for items is fixed at A Rupees/order
- The cost of deterioration:

$$C_D=cD_T=c\vartheta\left(\frac{aT^3}{6}+\frac{bT^4}{8}\right)$$

- The holding cost is a function of average inventory is given by:

$$C_H=ic\int_0^T I(t)dt=ic\left(\frac{aT^2}{2}+\frac{a\vartheta T^4}{12}+\frac{bT^3}{3}+\frac{b\vartheta T^5}{15}\right)$$

(Neglecting higher power of ϑ as ϑ is very small).

The net cost of the unpaid inventory at time t is the cost of the current inventory at any time t, minus the profit on the amount sold during time M, minus the interest earned from the sales revenue during time M. The extra amount that can be paid is determined by profit on the amount sold after the permissible delay time M. Therefore, the interest payable per cycle for the inventory not being sold after due date is given by:

$$P_T=I_p\int_M^P\left(cI(t)-(s-c)\int_0^M(a+bt)dt-sI_e\int_0^M(a+bt)tdt\right)dt-(s-c)I_p\int_0^{P-M}(a+bt)tdt$$

$$= I_p c\left[\begin{array}{l} a(P-M)\left\{\begin{array}{l} T\left(1-\dfrac{\vartheta(P^2+M^2+PM)}{6}\right)-\dfrac{P+M}{12} \\ \left(6-\vartheta(P^2+M^2)\right)+\dfrac{\vartheta T^3}{6} \end{array}\right\}+ \\ b\left\{\dfrac{T^2}{2}(P-M)-\dfrac{P^3-M^3}{6}\left(\dfrac{T^2}{2}-1\right)+\right. \\ \left.\dfrac{\vartheta(P^5-M^5)}{40}+\dfrac{\vartheta}{8}T^4(P-M)\right\}- \\ \left\{(s-c)\left(aM+\dfrac{bM^2}{2}\right)+sI_e\left(\dfrac{aM^2}{2}+\dfrac{bM^3}{3}\right)\right\}(P-M) \end{array}\right]$$
$$-(s-c)I_p\left(a\frac{(P\_M)^2}{2}+b\frac{(P\_M)^3}{3}\right)$$

Interest earned per cycle, I_T is the interest earned during the positive inventory is given by:

$$I_T = sI_e\left(\int_0^M (a+bt)t\,dt + \int_0^{T-P}(a+bt)t\,dt\right)$$
$$= sI_e\left[\frac{a}{2}\left(M^2+(T-P)^2\right)+\frac{b}{3}\left(M^3+(T-P)^3\right)\right]$$

The variable cost is aggregately comprised of ordering cost, carrying cost, cost due to deterioration of materials and the interest payable minus the interest earned. Thus, the total variable cost per cycle, C_{VT} is defined as:

$C_{VT} = A+C_D+C_H+P_T-I_T$

Substituting the values from above equations we obtain:

$$C_{VT}(P,T) = A+\frac{ca\vartheta T^3}{6}+\frac{cb\vartheta T^4}{8}+ic\left(\frac{aT^2}{2}+\frac{a\vartheta T^4}{12}+\frac{bT^3}{3}+\frac{b\vartheta T^5}{15}\right)$$
$$+cI_p\left[\begin{array}{l} a(P-M)\left\{T\left(1-\dfrac{\vartheta(P^2+M^2+PM)}{6}\right)-\right. \\ \qquad\left.\dfrac{P+M}{12}\left(6-\vartheta(P^2+M^2)\right)+\dfrac{\vartheta T^3}{6}\right\} \\ +b\left\{\begin{array}{l}\dfrac{T^2}{2}(P-M)-\dfrac{P^3-M^3}{6}\left(\dfrac{T^2}{2}-1\right) \\ +\dfrac{\vartheta(P^5-M^5)}{40}+\dfrac{\vartheta}{8}T^4(P-M)\end{array}\right\} \\ -\left\{(s-c)\left(aM+\dfrac{bM^2}{2}\right)+sI_e\left(\dfrac{aM^2}{2}+\dfrac{bM^3}{3}\right)\right\}(P-M) \end{array}\right]$$
$$-(s-c)I_p\left(\begin{array}{l} a\dfrac{(P-M)^2}{2} \\ +b\dfrac{(P-M)^3}{3}\end{array}\right)-sI_e\left[\begin{array}{l}\dfrac{a}{2}\left(M^2+(T-P)^2\right) \\ +\dfrac{b}{3}\left(M^3+(T-P)^3\right)\end{array}\right]$$

The variable cost per unit time is given by:

$$C_T(P,T)=\frac{C_{VT}(P,T)}{T}$$

The optimal values of P and T will provide the optimal solution which can be obtained by variation techniques

Numerical illustrations:

Considering A = Rupees 200, c = 40, s = 46, I_p = 0.18 and I_c = 0.15

Case I for M = 0, T* = 2.5 months and P* = 1.85 Months which gives C_r = Rupees 1850.53
Case II for M = 0.5 month T* = 2.75 and P* = 1.9 and gives C_r = 984

RESULTS

One of the objectives of this study is to develop an inventory model for perishable items having linear demand pattern. Then with the help of this model total inventory cost is obtained. The practical aspects of inventory management like opportunity cost and effect of permissible delay in payment is also considered.

The total cost obtained then can be used to obtain an average inventory cost, which can be optimized using calculus techniques.

Numerical illustrations proves the applicability of the suggested model

DISCUSSION

The model considered above is suited for items having variable deterioration rate, earlier models have considered items having constant rate of deterioration. This model can be used for items like fruits and vegetables whose deterioration rate increases with time. Demand pattern considered here is linear demand patterns, which can also be converted into constant demand pattern.

The suggested model can further be extended for fixed credit period as well as allowable shortages. This study will help retailers in deciding their optimal ordering quantity to have minimum inventory cost.

CONCLUSION

This study can help substantially retailers or buyers in deciding their payment time, considering the benefit of permissible delay in payments. In this study items having linear demand are considered with variable rate of deterioration. The cost price and selling price are different which increases the practicality of this model as most of the earlier models considered cost price and selling price of the articles same. This model can be further extended for items having quadratic demand or power demand. As well as effect of inflation and time value of money can also be considered. This study will act as a catalyst for the study of permissible delay in payments.

REFERENCES

1. Aggarwal, S.P. and C.K. Jaggi, 1995. Ordering policies of deteriorating items under permissible delay in payments. J. Operat. Res. Soc., 46: 658-662. http://www.jstor.org/pss/2584538
2. Bierman, H. and J. Thomas, 1977. Inventory decisions under inflationary condition. Dec. Sci., 8: 151-155.
3. Buzacott, J.A., 1975. Economic order quantities with inflation. Operat. Res. Q., 26: 553-558.
4. Chandra, M.J. and M.L. Bahner, 1985. The effect of inflation and time valur of money on some inventory systems. Int. J. Prod. Res., 23: 723-729. http://cat.inist.fr/?aModele=afficheN&cpsidt=8515393
5. Chang, H.Y. and C.Y. Dye, 1999. An EOQ model for deteriorating items with time varying demand and partial backlogging. J. Operat. Res. Soc., 50: 1176-1182. http://cat.inist.fr/?aModele=afficheN&cpsidt=1993340
6. Chung, K.J. and Y.F. Huang, 2004. Optimal replenishment policies for EOQ inventory model with limited storage capacity under permissible delay in payments. Operat. Res., 41: 16-34. http://direct.bl.uk/bld/PlaceOrder.do?UIN=154981078&ETOC=RN&from=searchengine
7. Covert, R.P. and G.C. Philip, 1973. An EOQ model for items with weilbull distribution deterioration. IIE Trans., 5: 323-326. http://www.informaworld.com/smpp/ftinterface?content=a780509777&rt=0&format=pdf
8. Ghare, P.M. and G.F. Schrader, 1963. A model for exponentially decaying inventories. J. Ind. Eng., 14: 238-43.
9. Goyal, S.K., 1985. Economic order quantity under conditions of permissible delay in payments. J. Operat. Res. Soc., 36: 335-338. http://www.jstor.org/stable/2582421
10. Jamal, A.M.M., B.R. Sarkar and S. Wang, 1997. An ordering policy for deteriorating items with allowable shortage and permissible delay in payment. J. Operat. Res. Soc., 48: 227-236.
11. Misra, R.B., 1979. A note on optimal inventory management under inflation. Naval Res. Logist., 26: 161-165.
12. Shah, Y.K., 1977. An order-level lot-size inventory model for deteriorating items. IIE Trans., 9: 108-112. http://www.informaworld.com/smpp/ftinterface?content=a780520916&rt=0&format=pdf
13. Sarbjit, S. and S.S. Raj, 2007. Lot sizing decisions under trade credit with variable demand rate under inflation. Ind. J. Math. Sci., 3: 29-38. http://www.serialspublications.com/
14. Su, C.T., L.I. Tong and H.C. Liao, 1996. An inventory model under inflation for stock-dependent demand consumption rate and exponential decay. Opsearch, 20: 99-106.
15. Teng, J.T., 2002. On the economic order quantity under conditions of permissible delay in payment. J. Operat. Res. Soc., 53: 915-918. http://www.jstor.org/stable/822919
16. Chang, J.T., C.T. Chang and S.K. Goyal, 2005. Optimal Pricing and ordering policy under permissible delay in Payment. Int. J. Prod. Econ., 97: 121-129. DOI: 10.1016/j.ijpe.2004.04.010