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AN INFORMATION-THEORETIC IMAGE QUALITY MEASURE: COMPARISON WITH STATISTICAL SIMILARITY

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ABSTRACT

We present an information-theoretic approach for structural similarity for assessing gray scale image quality. The structural similarity measure SSIM, proposed in 2004, has been successfully used and verfied. SSIM is based on statistical similarity between the two images. However, SSIM can produce confusing results in some cases where it may give a non-trivial amount of similarity for two different images. Also, SSIM cannot perform well (in detecting similarity or dissimilarity) at low peak signal to noise ratio (PSNR). In this study, we present a novel image similarity measure, HSSIM, by using information - theoretic technique based on joint histogram. The proposed method has been tested under Gaussian noise. Simulation results show that the proposed measure HSSIM outperforms statistical similarity SSIM by ability to detect similarity under very low PSNR. The average difference is about 20dB.

Keywords: Joint Histogram, Image Structural Similarity, Image Quality Assessment, Image Processing

1. INTRODUCTION

Image similarity measurement is a fundamental issue in real-world applications. Image quality metrics play a significant role in image processing. It can be used to adjust image quality and optimize parameters in many image processing applications such as enhancement, compression, restoration, etc. The image quality assessment aims to design methods for objective assessment of quality versus subjective human image quality evaluation (Wang *et al*., 2002).

A simple way to measure the similarity between two images is to calculate the Mean-Squared Error (MSE) between the reference image and the distorted version. MSE is easy to compute, however, it performs poorly in pattern recognition (Wang *et al*., 2004; Simard *et al*., 2002).

An important objective measure was proposed in 2004 by (Wang and Bovik, 2004), where image distortion was considerd as a combination of three kinds of distortion: Correlation, luminance and contrast.

In general, image similarity measures can be classsifed into two main directions: Statistical-based and information-theoretic based quality measures.

1.1. Statistical Measures

The above measure proposed by Wang *et al*. (2004) which was called SSIM, used distance

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covariance to measure the structural similarity based on statistical measurements such as mean and standard deviation as follows Equation 1:

$$\rho(x,y) = \frac{(2\mu_x\mu_{y+}C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \quad (1)$$

where, $\rho(x,y)$ is the structural SSIM measure between the image x (original or reference image) and the image y (corrupted image), μ_x, μ_y, σ_x^2 *and* σ_y^2 are the statistical means and variances of pixel values in images x and y, σ_{xy} is the statistical covariance between images x and y, while the constants C_1 and C_2 are given by $C_1 = (K_1L)^2$ and $C_2 = (K_2L)^2$, with K_1 and K_2 are small constants and L = 255 (the maximum pixel value).

Sheikh *et al*. (2006), proposed further study on quality assessment. In Sampat *et al*. (2009) an improvement of SSIM was presented based on wavelet coefficients which are extracted at the same locations in the same wavelet sub bands of the two images; and the new measure proved to be insensitive to small geometric variations.

Some limitations of Image Quality Measurement are presented in (Zhang *et al*., 2009).

SSIM gives high level of similarity for noise free condition while it fails when noise increases. In addition, if the two images are dissimilar, SSIM may give an amount of similarity (Hashim and Hussain, 2014). This is so because SSIM can't reveal all image structural properties, so we need to more specific measurements that are image-dependent. Hashim and Hussain (2014) proposed methods to determine a reliable similarity between any two images, similar or dissimilar. The methods are based on image-dependent properties, specifically the results of edge detection and segmentation methods, in addition to the image statistical properties; and tested under effects of Gaussian noise, impulse noise, as well as blur.

The above-mentioned similarity measures are all analysed based on statistical moments, which we will study in this study versus information-theoretic approach in order to test similarity.

1.2. Information-Theoretic Measures

The use of information-theoretic analysis in image processing is possible if we assume that images are 2D random variables.

The most common measure of information is the Shannon-Wiener entropy measure. The entropy H of a discrete random variable X with the values in the set $\{x_1, x_2, \ldots, x_n\}$ is defined as (Shannon and Weaver, 1967):

$$H(X) = -\sum_{i=1}^{n} p_i log p_i$$

Where:

$$p_i = p_r[X = x_i]$$

The definition of entropy for a single random variable can be extended to a pair of random variables.

The joint entropy H(X,Y) of a pair of discrete 2D random variables, which are images in our case, with a joint distribution p_{ij} is given by (Viola and Wells, 1995):

$$H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij} \log p_{ij}$$

The mutual information between two discrete random variables X and Y is defined as (Viola and Wells, 1995):

$$I(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij} log \frac{p_{ij}}{p_i p_j}$$

The mutual information between two random variables can be defined as the amount of information that one random variable can give about the other one. Hence, mutual information as defined above is a measure of the decrease in the entropy of one of these variables if we are given the other one. As such, it is possible to use mutual information as well as other information-theoretic measures (like the histogram) to define similarity measures.

The joint intensity histogram for two images would be spread (hence, giving high joint entropy) when the two images are not aligned, while it is compact (hence, giving low joint entropy) when the two images are aligned. The Mutual Information (MI) is proposed by various authors as per (Wells *et al*., 1996; Maes *et al*., 1997; Pluim *et al*., 2003). The overlap invariant Normalized MI (NMI) have been proposed in (Studholme *et al*., 1999).

Regional MI (RMI) has been proposed in (Studholme *et al*., 2006), while Conditional MI (CMI) has been proposed in (Loeckx *et al*., 2010). These methods depend on summing local MI for regions of the images, rather than finding the global MI. Klein *et al*.

(2007; 2009) proposed Localized MI (LMI); where random samples are taken from regions in very iteration.

Zhuang *et al*. (2011) proposed spatially-encoded MI, where pixels are assigned weights based on their spatial location instead of having equal weights.

Darkner and Sporring (2013) has proposed a unified approach for NMI.

In this study, we present an information-theoretic image similarity measure and show its superior performance versus the classical similarity measure SSIM under additive Gaussian noise with several ratios of signal to noise.

2. JOINT HISTOGRAM

The concept of multidimensional histogram has been investigated by several researchers in different application domains (Hill *et al*., 1993; Pass and Zabih, 1999).

The joint-histogram represents the joint-probability of two or more random variables, images in our case. For the two-dimensional joint-histogram, the pixel values of two images are used as the coordinate axis. Given two images A and B and pixel intensity values *i* and *j*, H_{ij} (A, B) represents the probability of a pixel to have the value *i* on the image *A* and *j* on the image *B*. Hence, each entry is the number of times an intensity *i* in one image corresponds to an intensity *j* in the other.

If the joint histogram is normalized, it becomes an estimate of the joint Probability Distribution Function (PDF) of intensities in the images (Kuczynski and Miko£Ajczak, 2003).

We can notice that under noise-free conditions, the joint histogram of two identical images will give a straight-line peak at i = j, while under noisy conditions this criterion is perturbed. Based on this approach, we designed a novel similarity measure as follows.

3. RATIONALE

We noticed that SSIM measure introduced in (Wang *et al*., 2004) gives good measure of similarity between two similar images; however, it fails when noise is significant (i.e., at low PSNR). We need an enhanced measure that can perform well under low PSNR. We have utilized joint histogram and combined it with the original histogram to get the enhanced measure HSSIM; also we tested SSIM and HSSIM under disruptive conditions like additive Gaussian noise.

4. THE PROPOSED IT MEASURE

The original design of structural similarity measure SSIM was based on image statistical properties as in (Wang *et al*., 2004). In this paper we focus on image information-theoretic properties, specifically the joint histogram and propose the following image-dependent measure. First, we propose an error estimate between an original (reference) image x and a noisy version y of it. The estimate is based on the diagonal symmetry of self-joint histogram H, which is just a 2D extension of the reference image histogram:

$$h(x)=\{h_i(x), i\in(1,2,\cdots,N)\},$$

about $i = j$ axis as follows:

$$H_{ij}(x,x)=H_{ji}(x,x)$$

The Information-Theoretic (IT) error estimate of joint histograms (relative to the original histogram) can be designed as follows Equation 2:

$$E(x,y)=\sqrt{\frac{\sum_i\sum_j\left[(H_{ij}-H_{ji})\frac{1}{h_i+c}\right]^2}{2L^2}}\cdots \quad (2)$$

where, c is a *very small positive constant, inserted mainly to avoid division by zero*. Note that:

$$E(x,y)\geq 0$$

A maximal error estimate E∞(x,y) can be considered when noise power is very high (very low PSNR). Then the above estimate can be normalized with respect to the maximal relative error estimate as follows Equation 3:

$$e(x,y)=E(x,y)/E_\infty(x,y) \quad (3)$$

The normalization process will ensure that:

$$0\leq e(x,y)\leq 1 \quad (4)$$

Based on the above error estimate an information-theoretic similarity measure (which we name as HSSIM) can be proposed as follows Equation 5 and 6:

$$\lambda(x,y)=1-e(x,y) \quad (5)$$

where:

$$0\leq \lambda(x,y)\leq 1 \quad (6)$$

As in the case of SSIM, HSSIM ranges between 0 (for dissimilar images) and 1 (for identical images). If we denote SSIM by ρ(x,y) then we have the i equality:

$$0 \le \lambda(x,y) \le 1$$

where, ρ(x,y) = 1 for identical images, while ρ(x,y) = 0 for totally different images.

5. THE TEST ENVIRONMENT

The proposed HSSIM measure have been tested under Gaussian noise, which is the main source of noise in many image processing systems.

Different kinds of images has been used: A human face, a landscape and a geometric shape.

6. RESULTS AND DISCUSSION

The above algorithms have been simulated using MATLAB. **Figure 1 to 3** show performance of SSIM and HSSIM using similar images under Gaussian noise. Note that:

$$0 \le \lambda(x,y), \rho(x,y) \le 1$$

For completely similar images we have:

$$\lambda(x,y) = \rho(x,y) = 1$$

while for totally different images we have:

$$\lambda(x,y) = \rho(x,y) = 0$$

To calculate similarity measure SSIM, an M × M window (M = 11) is used with a standard deviation of 1.5 (Wang *et al*., 2004). The constants $C_1 = (K_1L)^2$ and $C_1 = (K_2L)^2$ (K_1 and K_2 being small constants, L = 255) where chosen as in (Wang *et al*., 2004) as K_1 = 0.01 and K_2 = 0.03. Note that the performance of SSIM is insensitive to these constants (Wang *et al*., 2004; Wang and Bovik, 2004). The constant in Equation 1 is taken as $c = 1\times10^{-15}$.

We also implemented the joint Histogram-based measure (HSSIM) as per Equation 4. E∞(x,y) is calculated at PSNR = -100 dB (total noise).

6.1. Performance under Gaussian Noise

After we implemented the joint histogram-based measure (HSSIM) as per Equation 4 and SSIM as per Equation 1, we tested their performance of detecting similarity under noisy conditions, specifically when the other image is corrupted with Gaussian noise. Peak signal to noise ratio (PSNR) was used in this test as follows:

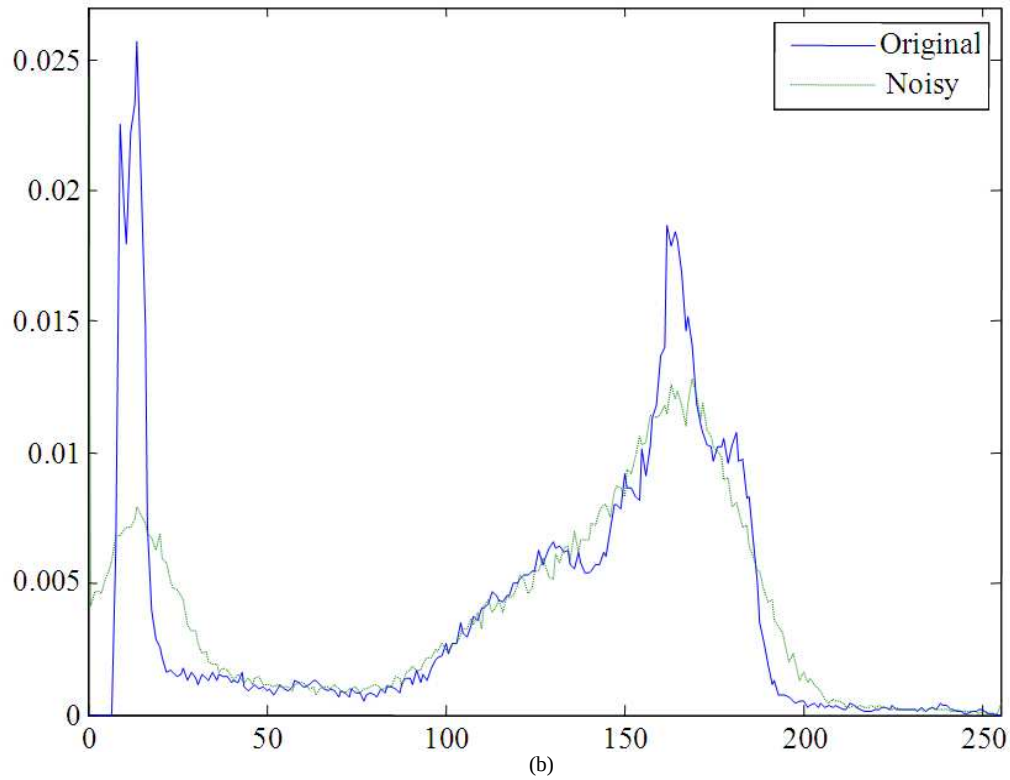
$$PSNR = \frac{L^2}{p_n}$$

where, p_n is the Gaussian noise variance (power).

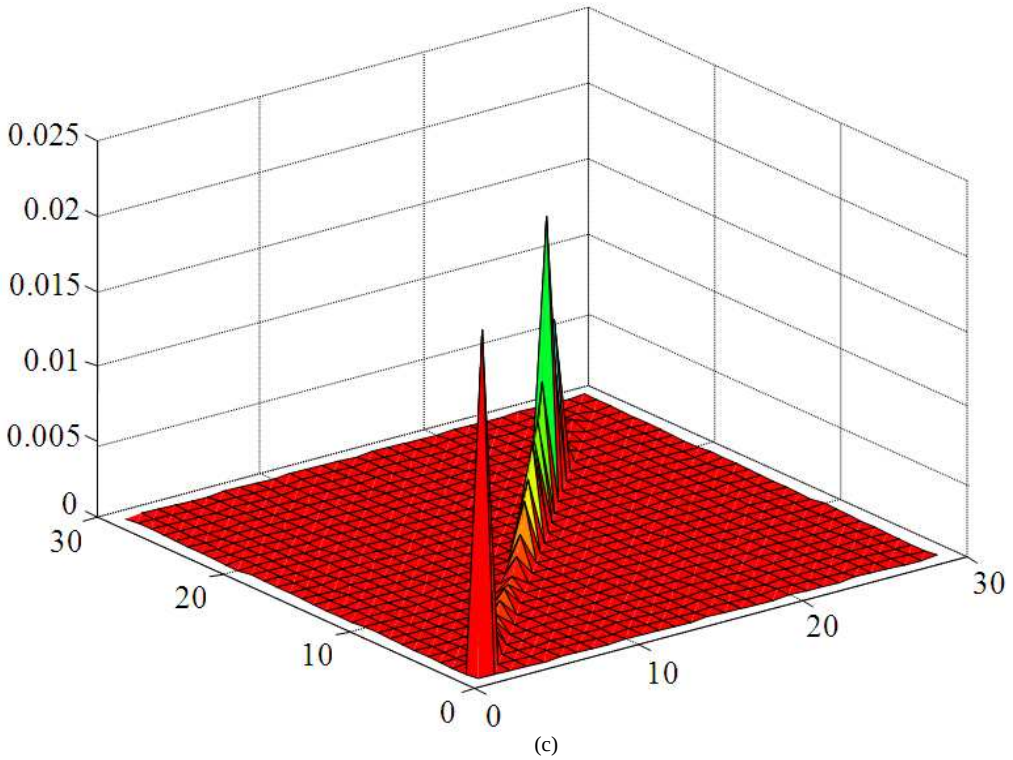
A maximal error estimate can be considered when noise power is very high with PSNR = 100 dB.

The result of using two similar images (a reference image and a noisy version of it) is shown in **Fig. 1 to 3** for various kinds of images: Landscape, human face and geometric shape.

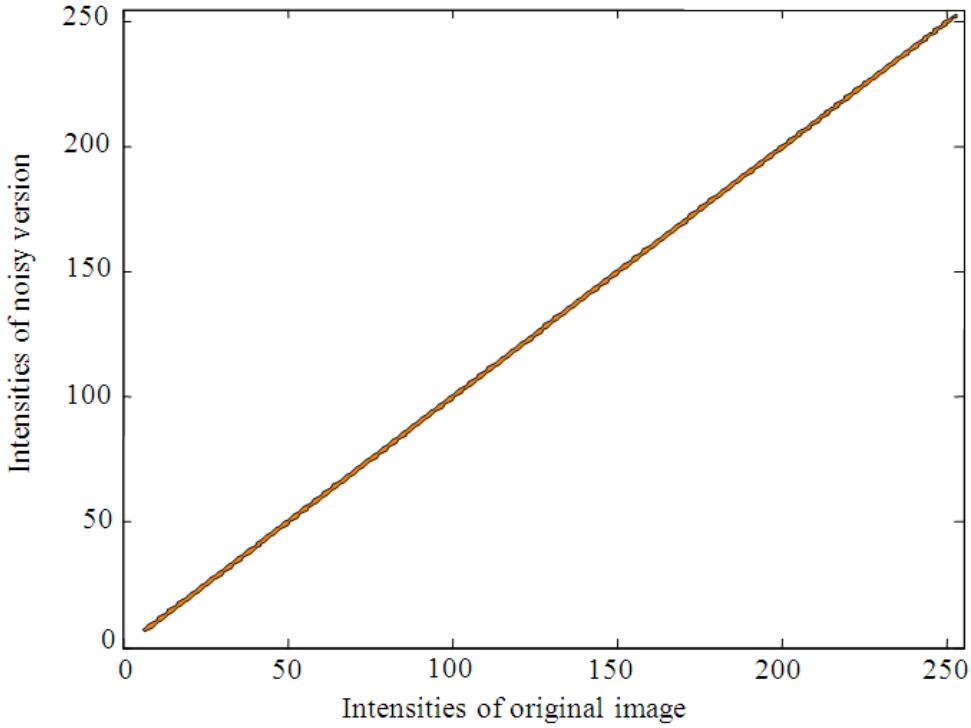
(a)


Original
Noisy
0.025
0.02
0.015
0.01
0.005
0
0
50
100
150
200
250

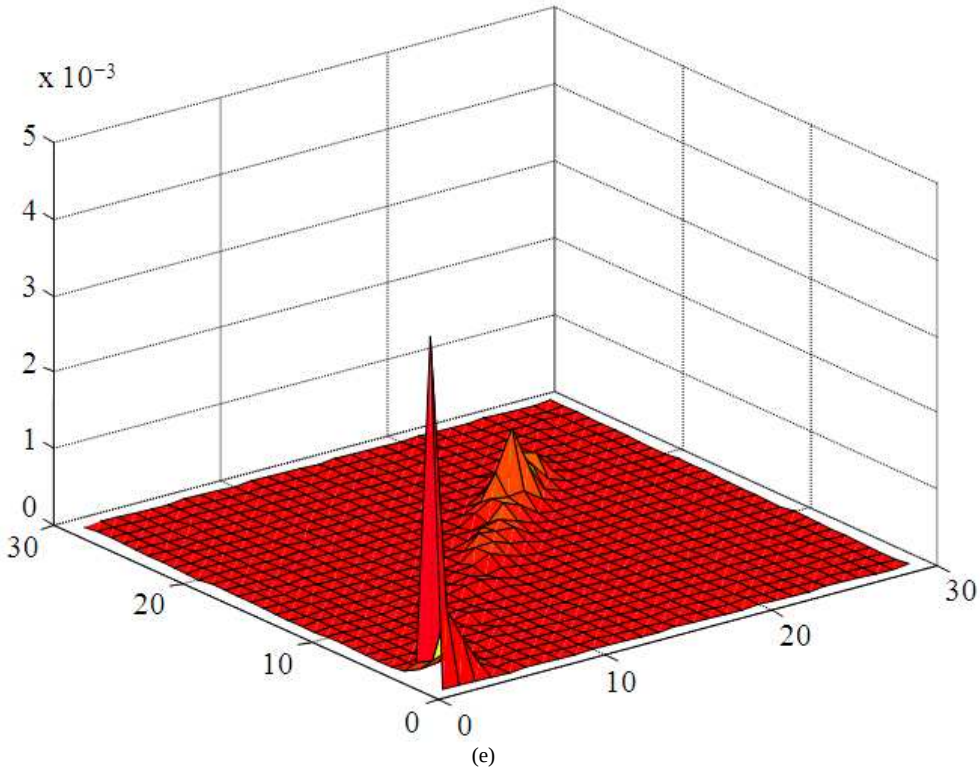
(b)


0.025
0.02
0.015
0.01
0.005
0
30
20
10
0
0
10
20
30

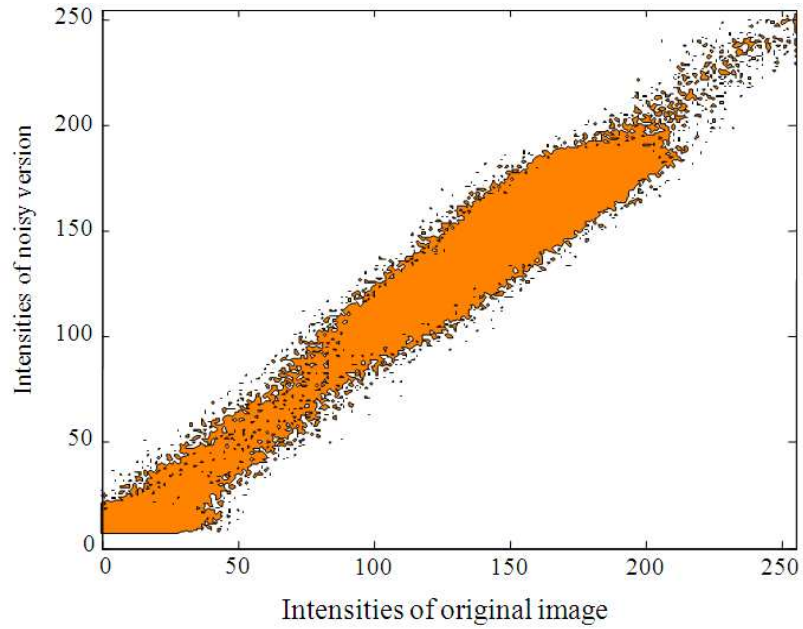
(c)


250
200
150
100
50
0
0
50
100
150
200
250
Intensities of noisy version
Intensities of original image

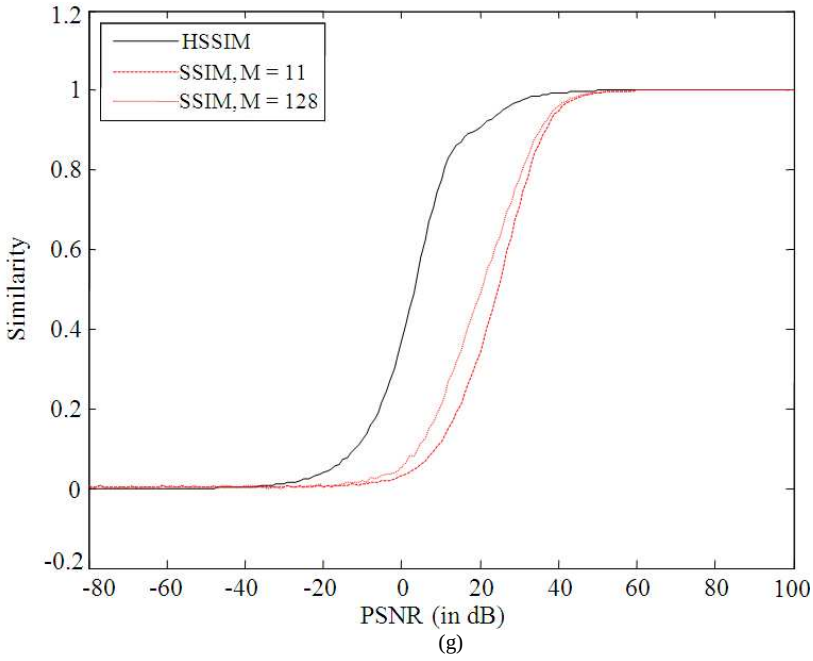
(d)


x 10−3
5
4
3
2
1
0
30
20
10
0
0
10
20
30

(e)

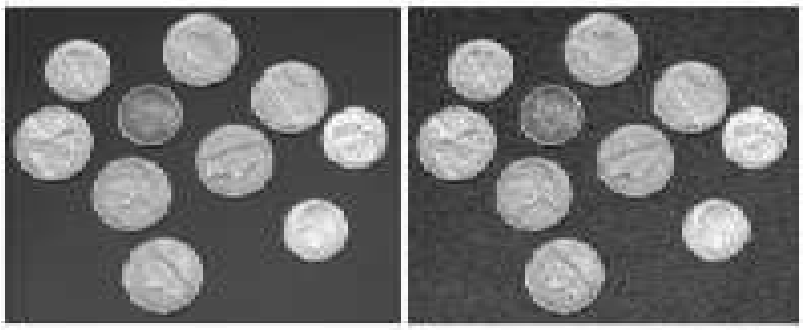


(f)

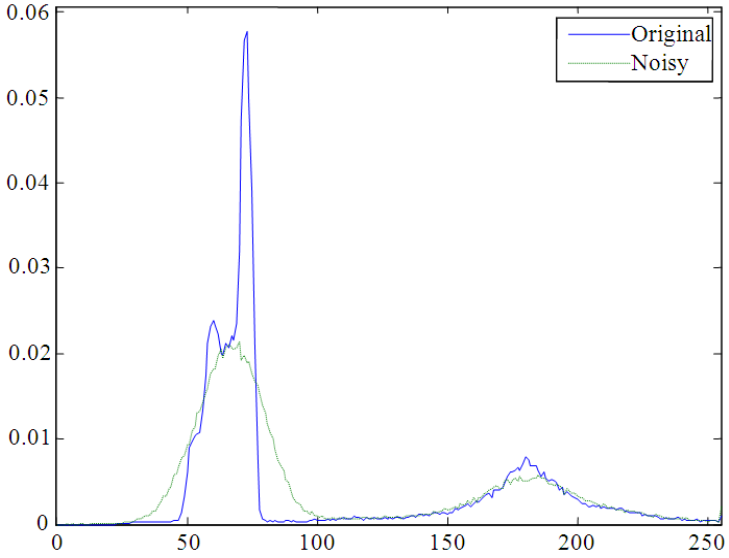


(g)

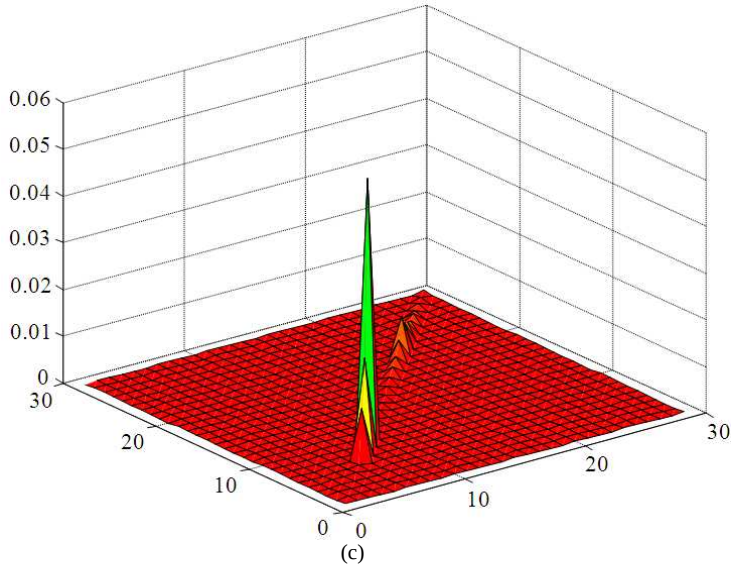
Fig. 1. Performance of HSSIM and SSIM using an image of landscape under Gaussian noise, (a) The test images (MATLAB Cameraman), (b) Histograms of original and noisy images, (c) Joint histogram of identical images (noise-free), (d) Contour plots for joint histogram of identical images (noise-free): Almost a straight line around i = j, (e, f) Joint histogram and its contour of original and noisy images. Symmetry can be detected around i = j, (g) Similarity vs. PSNR (dB): It is clear that HSSIM outperforms the conventional SSIM in its capability to detect similarity at low PSNR (almost 20 dB difference). Note that increasing the processing window length M up to half image width can improve SSIM performance a bit (dotted curve); however, further increase of M could give messy results



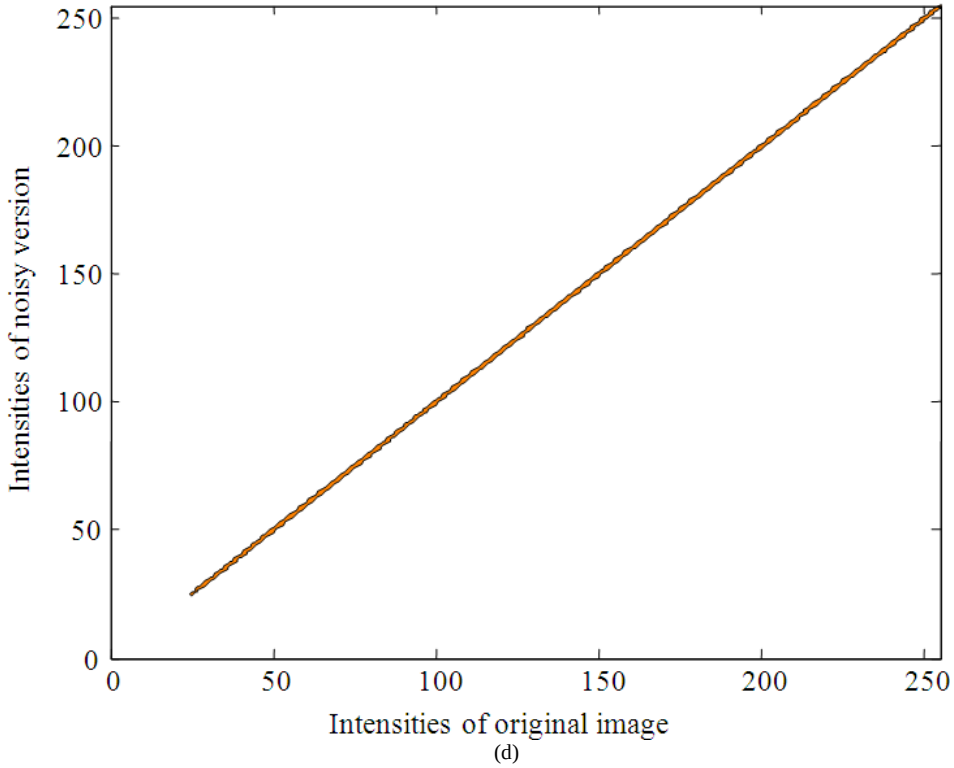
(a)


Original
Noisy
0.06
0.05
0.04
0.03
0.02
0.01
0
0
50
100
150
200
250

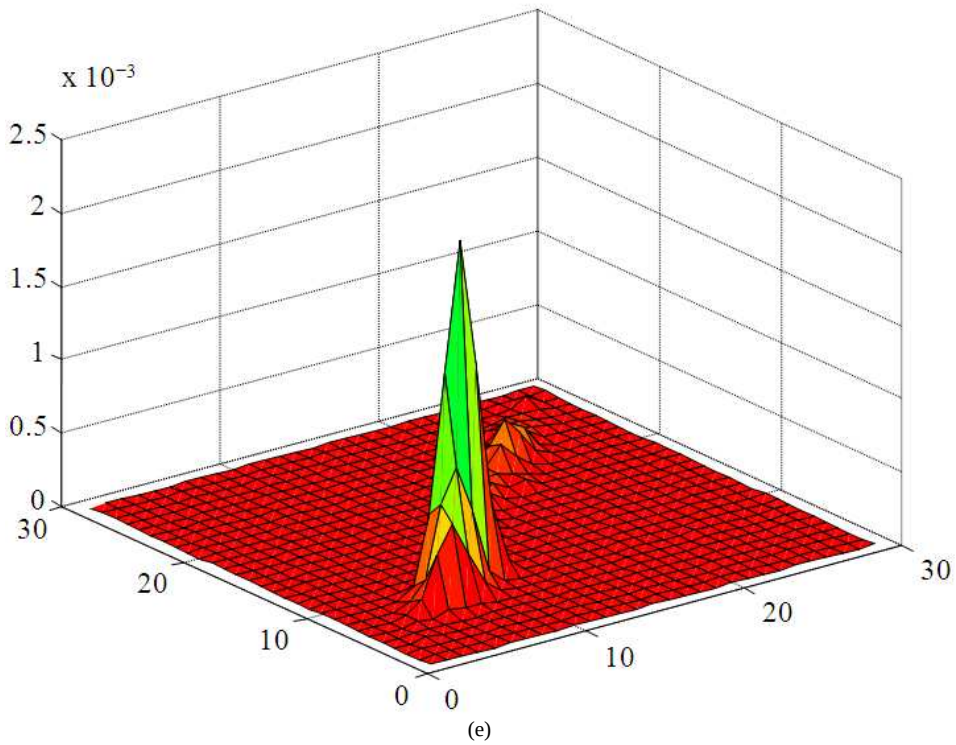
(b)


0.06
0.05
0.04
0.03
0.02
0.01
0
30
20
10
0
0
10
20
30

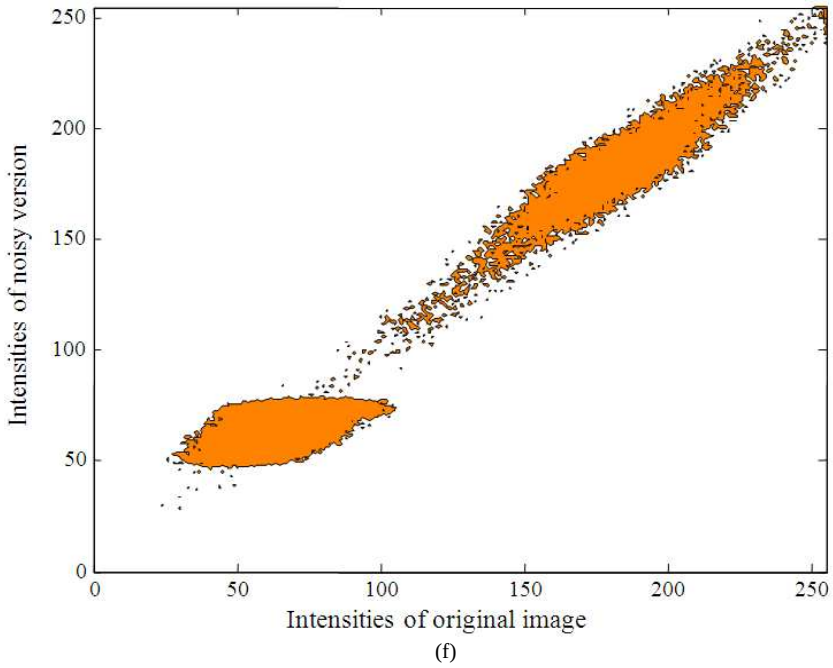
(c)


Intensities of noisy version
Intensities of original image
0
50
100
150
200
250

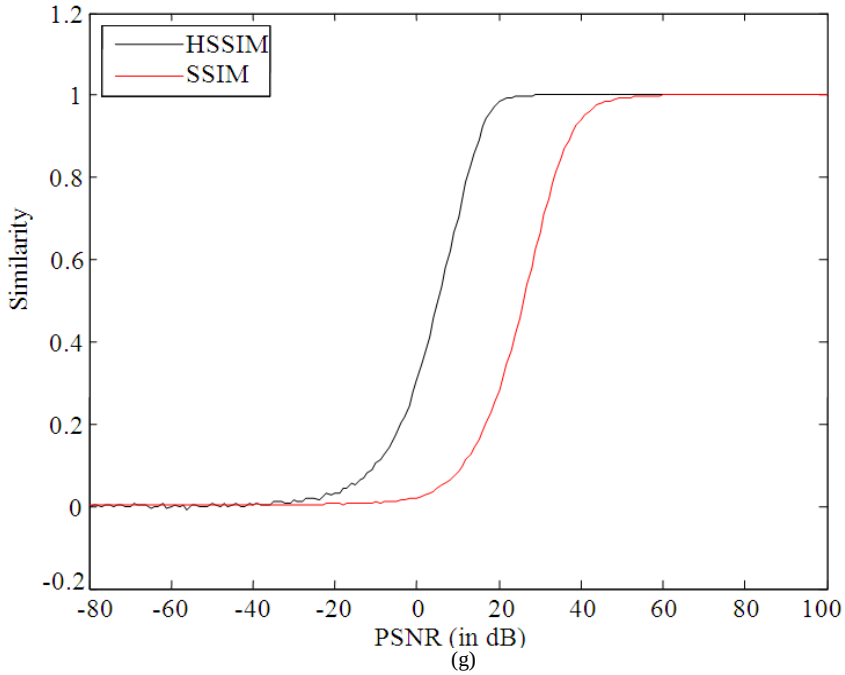
(d)


x 10⁻³
2.5
2
1.5
1
0.5
0
30
20
10
0
0
10
20
30

(e)



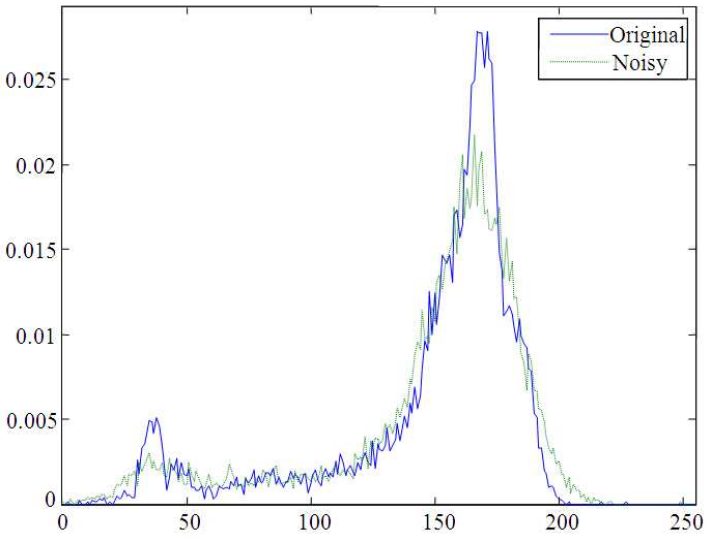
(f)



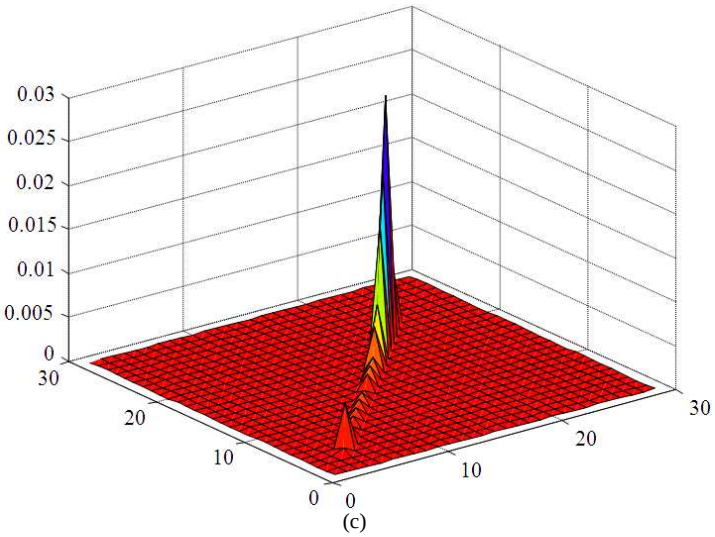
(g)

Fig. 2. Performance of HSSIM and SSIM (M = 11) for geometric images (taken from MATLAB). Note that HSSIM still outperforms the conventional SSIM in its capability to detect similarity at low PSNR (almost 20 dB difference). Compare with Figure 1, (a) Original Image, Noisy Image, PSNR (dB) = 28, (b) Histograms: Original solid; noisy dotted, PSNR (dB) = 28, (c) Joint Histogram: Identical images, PSNR (dB) = infinity, (d) Jt. Hist. (Contour): Identical images, PSNR (dB) = infinity, (e) Joint Histogram: Original image and noisy version, PSNR (dB) = 28, (f) Jt. Hist. (Contour): Original and noisy image, PSNR (dB) = 28, (g) Similarity vs. PSNR(dB)

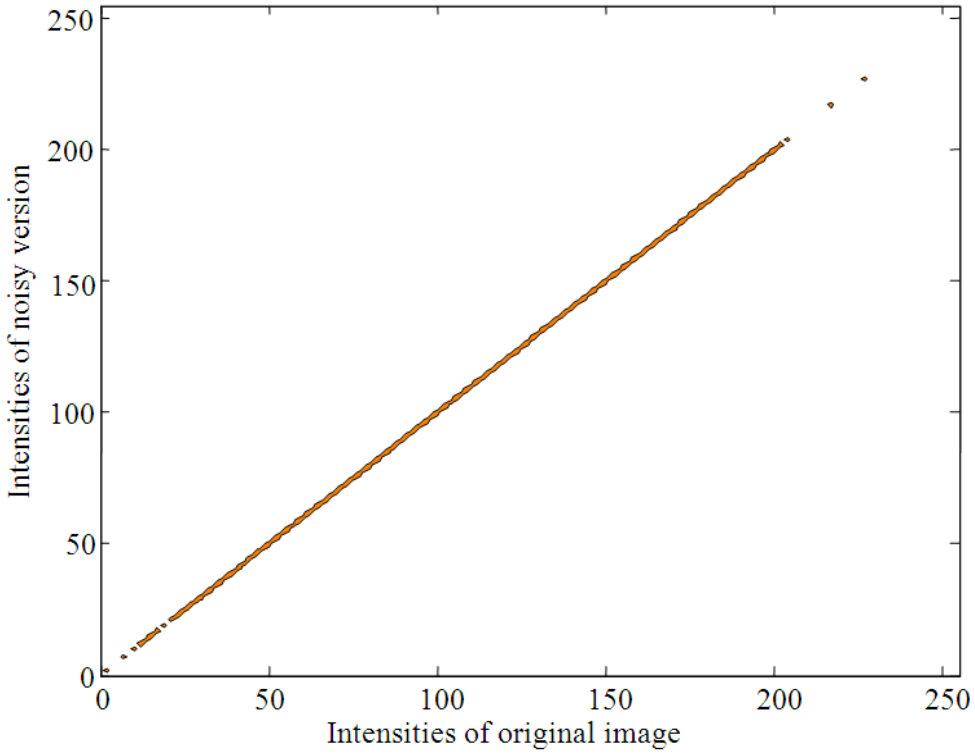
(a)


Original
Noisy
0.025
0.02
0.015
0.01
0.005
0
0
50
100
150
200
250

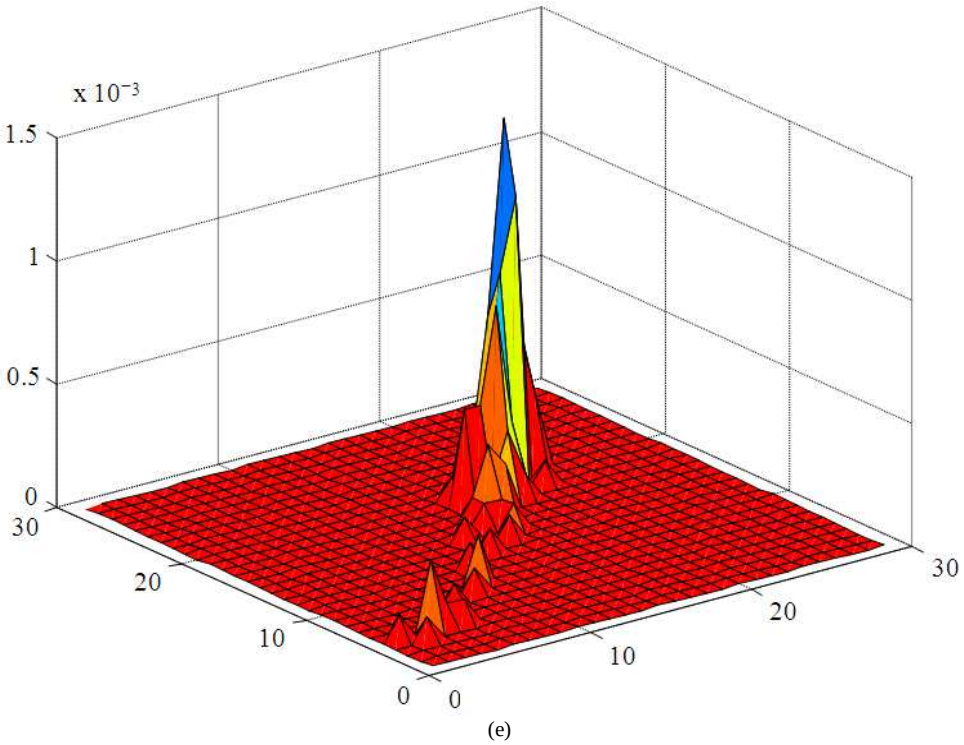
(b)


0.03
0.025
0.02
0.015
0.01
0.005
0
30
20
10
0
0
10
20
30

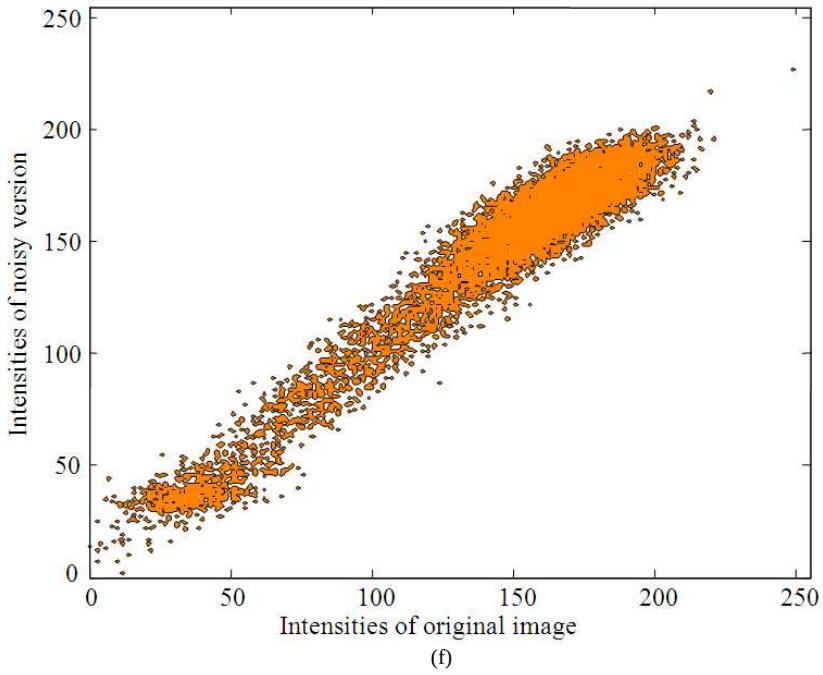
(c)


Intensities of noisy version
Intensities of original image
0
50
100
150
200
250

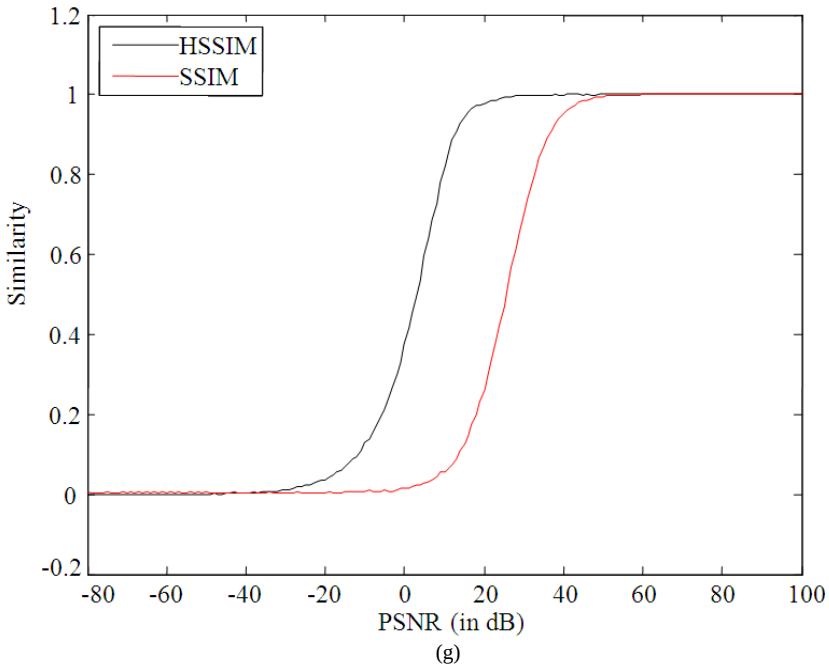
(d)


x 10⁻³
1.5
1
0.5
0
30
20
10
0
0
10
20
30

(e)



(f)



(g)

Fig. 3. Performance of HSSIM and SSIM (M=11) for a human face (taken from AT and T Database). Note that HSSIM still outperforms the conventional SSIM in its capability to detect similarity at low PSNR (almost 20 dB difference; comparable with Figures 1 and 2), (a) Original Image, Noisy Image, PSNR (dB) = 28, (b) Histograms: Original solid; noisy dotted, PSNR (dB) = 28, (c) Joint Histogram: Identical images, PSNR (dB) = infinity, (d) Jt. Hist. (Contour): Identical images, PSNR (dB) = infinity, (e) Joint Histogram: Original image and noisy version, PSNR (dB) = 28, (f) Jt. Hist. (Contour): Original and noisy image, PSNR (dB) = 28, (g) Similarity Vs. PSNR(dB)

We used the images "cameraman" and "coins" from MATLAB and a human face from AT and T face database (DOF, 1992).

The performance of HSSIM as compared to SSIM (represented by Equation 1) is shown in **Fig. 1 to 3**, which include a test under Gaussian noise. These results show about 20 dB difference of capability for HSSIM over SSIM to detect similarity at low PSNR.

6.2. Effects of Analysis Parameters

We noticed that the parameter c in the proposed error estimate (as per Equation 1) has no significant effect as long as it is kept small. Also, the choice of the window length M has no significant effect as long as it is kept at reasonable values for local treatment as per (Wang and Bovik, 2004). Choosing a larger window length can improve to some extent the SSIM performance as shown in **Fig. 1**, where M is varied up to half the image width. This is due to the fact that increasing the analysis window length may add more correlative information. However, increasing M beyond that value may result in degrading SSIM performance, where it gives messy result for M = image width.

7. CONCLUSION

A novel information-theoretic, image-dependent quality assessment measure has been proposed and tested versus structural similarity measure (SSIM) under Gaussian noise. The measure is based on joint histogram. Note that no interpolation has been used in this paper. Using interpolation can further improve performance, however, it would be the focus of a future work. It is shown that the proposed measure HSSIM out performs the conventional SSIM by almost 20 dB difference of PSNR in noisy environments for various kinds of images: Landscape, human face and geometric images. Increasing the window length of SSIM can improve its performance a bit; however, a significant increase may degrade its performance.

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9. ADDITIONAL INFORMATION

9.1. Funding Information:

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9.2. Author's Contributions

Asmhan Flieh Hassan: She is the main contributor, as this paper is part of her PhD program at the School of Mathematics and Statistics, HuaZhong Normal University, China. She contributed to preparation, analysis and algorithmic simulation.

Dong Cai-lin: He is the First Supervisor from School of Mathematics and Statistics, HuaZhong Normal University, China. He contributed to the preparation, mathematical analysis, and review of the paper.

Zahir M. Hussain: He is the Second Supervisor, from the Faculty of Computer Science and Mathematics, University of Kufa, Najaf, Iraq; also a Professor (adjunct) at the School of Engineering, Edith Cowan University, Australia. He contributed to preparation, analysis, computer simulations, and review of the paper.

9.3. Ethics:

The Authors utilized the standard AT&T face database in this research, which is an internationally acknowledged database. All analyses and simulations in this work have been performed by the Authors. To the best of the Author's knowledge, this work is clear of any ethical issues.

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