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A Cartesian Regulator for an Ideal Position-Servo Actuated Didactic Mechatronic Device: Asymptotic Stability Analysis

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Abstract: Position-servo actuators are by themselves feedback mechatronics systems modeled by Ordinary Differential Equations (ODE). From a technological point of view, position-servos are based upon an electrical motor, a shaft angular position sensor, and a dominant Proportional controller. These position servo actuators are at the core of several real-life practical and didactic mechatronics and robotics systems. The contribution of this study is the introduction of a novel position regulator in Cartesian space and the stability analysis of a real-world mechatronic system involving the following mechatronics ingredients: A position servo actuated pendulum endowed with position sensing for feedback and a novel nonlinear integral controller for direct position regulation in Cartesian space avoiding the inverse kinematics computational burden. Because of the nonlinear nature of the control system, the standard analysis tools from classic linear control cannot be utilized, thus this study invokes Lyapunov stability arguments to prove asymptotic stability and to provide an estimate of the domain of attraction.

Keywords: Actuators, Position Servo, Pendulum, Control, Stability, Domain of Attraction, Nonlinear Systems, Differential Equations, Robotics

Introduction

Control systems of real-world processes require actuators to handle the process to be controlled. Mechatronics and robotic devices are also equipped with actuators that translate low-power control input signals into system motion. So, the position servo actuators are electromechanical devices that take the energy-electricity and turn it into a motion (Sima and Zapciu, 2022).

The description of a modular mechatronic system for didactic purposes is presented in (Petrescu *et al*., 2018).

On the other hand, position servo actuators are at the core of many mechatronics and low-scale robotics systems such as Robot arms, humanoids-biped mobile robots- and wheeled or legged mobile robots.

Acording to Hasan and Dhingra (2020): "Servo motors refers to a complete system that includes a motor itself, driver, motor position/velocity sensor that runs based on some close loop control algorithm".

A low-cost 2 Degrees Of Freedom (DOF) servo actuated drawing robotic arm is presented (Fahim *et al*., 2019).

This study focuses, from an automatic control point of view, upon a low–cost didactic position servo actuated mechanism, which involves the following mechatronics ingredients: Position servo actuator, external position sensor (incremental encoder), pendular mechanism, and a novel control system (Fig. 1 for setup hardware of system under study).

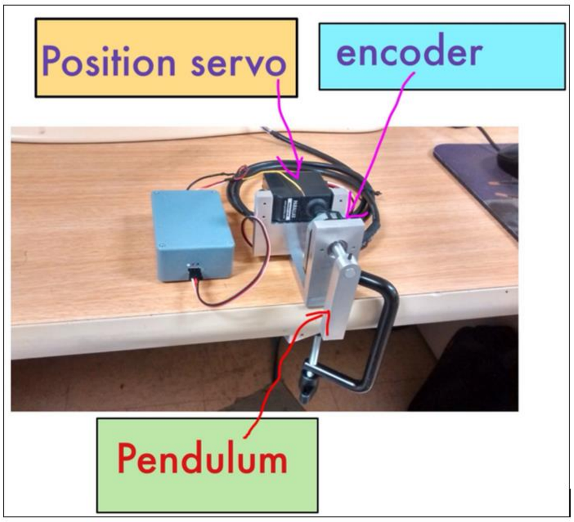

Fig. 1: A position-driven pendulum experimental setup powered by a position servo actuator (at Robotics Lab., CICESE)

Mathematical models of automatic control systems are usually described by differential equations (Astrom and Murray, 2008), so the analysis of automatic control systems resorts to differential equations tools, mainly from Lyapunov stability theory.

This study introduces the mathematical model of a real-life automatic control system: A position–servo commanded pendulum (Fig. 1) under feedback control of a novel Cartesian regulator which yields a whole closed-loop system mathematical structure modeled by the nonlinear ODE:

$$\frac{d}{dt}z \triangleq \dot{z} = -kl\left[1-\left(\frac{z-yd}{l}\right)^2\right]z, \tag{1}$$

where, $z \in \mathbb{R}$ stands for the state variable-Cartesian position error-and $l>0$, $k>0$, $y_d \in \mathbb{R}$ are real parameters.

For the concept of asymptotic stability and the direct Lyapunov's stability method, the reader is referred to one of the standard textbooks on the topic such as (Khalil, 2002; Hirsch and Smale, 1974; Hale, 1980; Vidyasagar, 1993) for a thorough treatment.

Throughout this study, the scalar variable $t \geq 0$ stands for the independent variable (time), $\mathbb{R}$ denotes the set of real numbers which are expressed by italic small letters and occasionally, by small Greek letters. The *n*-dimensional real vector space is denoted by $\mathbb{R}^n$ whose entries are n×1 column format of vectors: $x = col(x_1, x_2, \cdots, x_{xn}) \in \mathbb{R}^n$. Vectors are denoted by bold small letters, either Latin or Greek. Super–index ^{T}: $(\cdot)^T$ stands for vector transposition.

These papers resort to an asymptotic stability tool adapted here for autonomous ODEs as the following corollary inspired from Theorems 4.1 and 4.9 of the textbooks (Khalil, 2002):

Corollary 1. Asymptotic Stability and Estimate of Domain of Attraction

Consider the autonomous differential equation:

$$\dot{x} \triangleq \frac{dx}{dt} = f(x), \tag{2}$$

where, $f(x)$ is assumed to be locally Lipschitz from a domain $D \subset \mathbb{R}^n$ into $\mathbb{R}^n$ containing $x = 0 \in \mathbb{R}^n$, Suppose that the origin $x = 0 \in D$ is an equilibrium point of Eq. 2 (i.e., $f(0) = 0$).

Let $z(x)$: D → R a continuously differentiable function such that:

$$w_1(x) \leq z(x) \leq w_2(x), \tag{3}$$

and:

$$\frac{dz}{dt} = \frac{dz^T}{dx} f(x), \leq -w_3(x) \leq 0, \tag{4}$$

$\forall x \in D$, where $w_1(x)$ $w_2(x)$ and $w_3(x)$ are continuous positive definite functions in D.

Then $x = 0$ is a locally asymptotically stable equilibrium. Moreover, if r and c are chosen such that:

$$B_r \triangleq \{\|x\| \leq_r\} \subset D, \tag{5}$$

and:

$$c \triangleq \min_{\|x\|} = r\{w_1(x)\}, \tag{6}$$

then, every trajectory of Eq. 2 starting in:

$$A \triangleq \{x \in B_r \,|\, w_2(x) \leq c\} \tag{7}$$

is bounded and it satisfies $x(t) \to 0$ as $t \to \infty$.

A closely related result about boundedness and convergence set is presented as Theorem 5.4 in the textbook by Astrom and Wittenmark (1995), (pg. 204).

According to the Lyapunov theory, function $z(x)$ plays the role of a Lyapunov function, so allowing the use of a powerful mathematical machinery-Lyapunov stability theorems-to study stability of the system.

For a choice of r in the definition of Eq. 5 for set B_r - hypersphere of radius r centered at the origin, all functions: w_1, w_2, and w_3 are required to be known to look for the largest hypersphere B_r strictly inscribed into D where these functions w_i are positive definite ones.

Concerning constant *c*, the definition of Eq. 6 about the computation of constant c means that it is computed as the minimum value of function $w_1(x)$ evaluated for all x such that $\|x\| = r$.

This study borrows the definition of region of attraction from (Vidyasagar, 1986), namely.

The region of attraction is defined as the set S of initial conditions $x_0 = x(0) \in D$ which have the property that any solution trajectories starting from them eventually approach the origin (assumed to be an asymptotically stable equilibrium), i.e.:

$$S \triangleq \{x_0 \,|\, x(t) \to 0 \, as \, t \to \infty\}$$

Regions of attraction are (Vidyasagar, 1986): Invariant sets of system Eq. 2 and they are open sets.

An estimate of the domain of attraction, say *A*, is a proper subset of the true and unique Region of Attraction S.

It is worthy of remark that set A defined in Eq. 7 can be thought of as an estimate of the domain of attraction.

Plant Model and Control Objective

A mathematical model of n Degrees of Freedom (DOF) position-servo actuated mechanisms-robot manipulators included- has the general structure:

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$$f\left(t,q,\dot{q},\ddot{q},u,\tau_d\right)=0, \quad (8)$$

$$y=h\left(t,q,\dot{q},\ddot{q},u\right), \quad (9)$$

where, *f* and *h* are smooth functions. $q,\dot{q},\ddot{q}\in\mathbb{R}^n$ stand for the generalized positions, velocities, and accelerations, respectively. Variables $u,y,\tau_d\in\mathbb{R}^n$ are the mechanism input, output, and exogenous disturbances, respectively.

Since this study deals with models of ideal position servos and ideal position-servo actuated mechanisms, a few definitions can be introduced.

Definition 1. Ideal Position-Servo

By ideal position-servo, this study refers to position-servos having the following properties:

- Output variable: Shaft position *q*
- Memoryless (algebraic equation model, no ODE model)
- Identity input-output mapping
- Disturbance free
- Without any constraint (neither position nor speed nor torque)

The input-output relationship (Fig. 2) of an ideal position-servo is modeled by:

$$q=u, \quad (10)$$

where, *u* is the servo input variable -desired shaft position.

One recognizes that the ideal servo model of Eq. 10 may be the simplest and strongly imperfect one. An alternative and more realistic dynamic model of position servo actuators has been introduced by Urrea and Kern (2016).

Definition 2. Ideal Position-Servo Actuated Mechanisms

Ideal position-servo actuated mechanisms (also called position commanded mechanisms or position-driven ones) are those whose joints are equipped with ideal position servo actuators.

For a model of position-servo actuated robot arms, function f in Eq. 8 may be obtained from the so-called robotics jargon: The robot dynamic model (Kelly *et al.*, 2005, Ch. 3), whereas function h in Eq. 9 is closely related to the robot kinematics map (Spong *et al.*, 2020, Ch. 3).

For 'Ideal position-servo actuated mechanisms' governed by Eq. 8 and 9 and due to the ideal position servo actuator model of Eq. 10, then, the particular structure in Eq. 8 becomes:

$$f\left(t,q,\dot{q},\ddot{q},u,\tau d\right)=q-u \quad (11)$$

Plant Model: *Ideal Position-Servo Actuated Mechanism*

This study focuses upon the following innocuous mechanism: The Ideal position-servo actuated pendulum depicted in Fig. 3 which is assumed to be modeled by:

$$q=u, \quad (12)$$

$$y=-l\cos \quad (13)$$

where, *u*, *y* ∈ *R* are the input and output of the plant, respectively, *q* stands for the angular position of the rod concerning the downward vertical axis, and l > 0 is the rod length. System output y is the variable to be controlled which has the physical meaning of pendulum Cartesian position-vertical or perpendicular distance from the horizontal axis X to pendulum tip as shown in Fig. 3. It is assumed that the rod mass is negligible and its pivot is frictionless. Equation 12 is because an ideal position-servo actuator has been assumed (Definition 1). From an automatic control viewpoint, this physical plant is a Single Input-Single Output (SISO) nonlinear static-memoryless system: y = −lcos(u).

It is assumed that variables q and y are available from measurement. Also, parameter l is assumed to be known. So all of them may be utilized by the controller.

Control Objective: Cartesian Regulation

Let us define the Cartesian position error $\tilde{y}\in\mathbb{R}$ as:

$$\tilde{y}\triangleq y_d-y, \quad (14)$$

where the arbitrary user-selected desired output y_d is assumed to be constant and to satisfy:

$$\left|y_d\right|<l.$$

The Cartesian position regulation control objective is:

$$\lim_{t\to\infty}\tilde{y}\left(t\right)=0. \quad (15)$$

Proposed Controller

The proposed smooth dynamic controller to compute the control action *u* to be sent to the position servo actuator is modeled by:

$$\dot{u}=k\sin\left(q\right)\tilde{y}, \quad (16)$$

where, *k* > 0 is a user-free parameter. So, the regulator 'parameters' are *k* > 0 and the initial condition *u*(0) ∈ ℝ to be chosen by the user.

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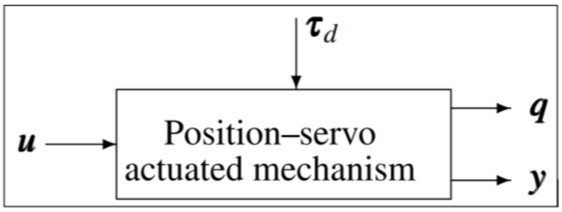

Fig. 2: Input-Output sketch of position-servo actuated mechanisms

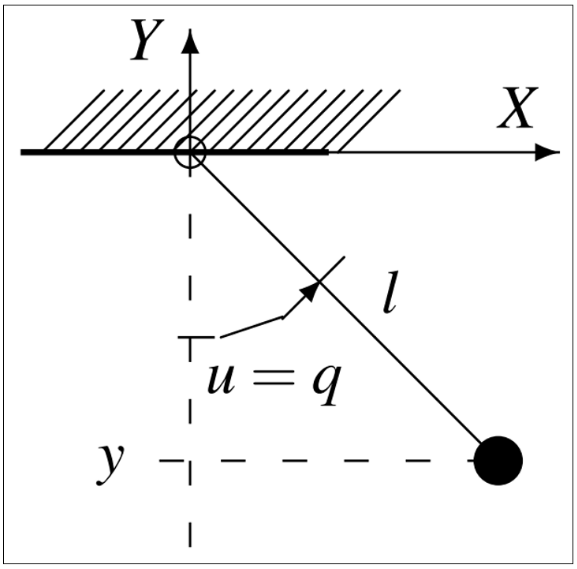

Fig. 3: Skeleton sketch of the position-servo actuated pendulum

A noticeable practical feature of the proposed control law in Eq. 16 is that neither velocity measurement nor velocity estimation is needed for its implementation.

Analysis

Isolating variable q from plant model Eq. 13, it results:

$$q = \arccos\left(-\frac{y}{l}\right). \tag{17}$$

The time derivative of the output y in Eq. 13 is:

$$\dot{y} = 1\sin(q)\dot{q}, \tag{18}$$

$$= l\sin(q)\dot{u}. \tag{19}$$

Model of the Closed-Loop System

The closed–loop system is obtained by substituting the derivative u˙ of the control action u from the control law Eq. 16 into 19:

$$\dot{y} = kl\sin^2(q)\tilde{y},$$

which thanks to the definition in Eq. 14 and assumption on constantly desired output $y_d\left(\Rightarrow \dot{y} = -\dot{\tilde{y}}\right)$, can be rewritten as:

$$\dot{\tilde{y}} = -kl\sin^2(q)\tilde{y}.$$

This equation can be better written as an autonomous one by substituting q from Eq. 17 and *y* from Eq. 14 these yields:

$$\dot{\tilde{y}} = -kl\sin^2\left(\arccos\left(\frac{\tilde{y} - y_d}{l}\right)\right)\tilde{y}. \tag{21}$$

An equivalent form can be obtained by using the following relation:

$$\sin(\arccos(x)) = \sqrt{1 - x^2},$$

which holds as far as |x| ≤ 1.

Thus, the closed-loop differential Eq. 21 becomes:

$$\dot{\tilde{y}} = \underbrace{-kl\left[-\left(\frac{\tilde{y} - y_d}{l}\right)^2\right]}_{\varphi(\tilde{y}, y_d)}\tilde{y}, \tag{22}$$

which is valid for:

$$\tilde{y} \in \left\{\left|\frac{\tilde{y} - y_d}{l}\right| \le 1\right\},$$

in other words, for y satisfying:

$$-l + y_d \le \tilde{y} \le l + y_d.$$

Let the domain D be defined as:

$$D \triangleq \left\{\tilde{y} \in \mathbb{R} \middle| -l + y_d < \tilde{y} < l + y_d\right\}, \tag{23}$$

where differential Eq. 22 under analysis is defined. Such a differential Eq. 22 describes the closed–loop system behavior in function of the position error y, so in control systems analysis, it is also so-called the error model system or equation.

Straightforward substitution of numerical values from Table 1 into Eq. 23 yields:

$$D = \left\{\tilde{y} \in \mathbb{R} \middle| -0.5 < \tilde{y} < 1.5\right\}. \tag{24}$$

This is the domain where the nonlinear differential Eq. 22 under numerical parameters in Table 1 makes sense. The system Eq. 22 equilibria are the solution y of:

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$$\varphi(\tilde{y}, y_d) = 0.$$

Although the origin y = 0 is a solution, two more solutions are:

$$\tilde{y} = l + y_d,$$

and:

$$\tilde{y} = -l + y_d$$

Notwithstanding, the unique solution within the domain D is the trivial one $y = 0$.

In sum, the above arguments are proof of the following:

Proposition 1. Closed–loop system ODE

Consider the plant model Eq. 12 and 13 under control of the proposed Cartesian regulator of Eq. 16. Then, the whole closed–loop system behavior is modeled by the autonomous nonlinear Ordinary Differential Eq. 22:

$$\dot{\tilde{y}} = -kl\left[1 - \left(\frac{\tilde{y} - y_d}{l}\right)^2\right]\tilde{y},\ \tilde{y} \in D. \tag{25}$$

being the origin y = 0 its unique equilibrium within D is defined in Eq. 23.

Stability Analysis

For the reader's convenience let us rewrite explicitly the right-hand side of Eq. 22:

$$\begin{aligned}
\varphi(\tilde{y}, y_d) &\triangleq -kl\left[1 - \left(\frac{\tilde{y}^3 - 2y_d + y_d{}^2}{l^2}\right)\right]\tilde{y}, \\
&= kl\left[\tilde{y} - \left(\frac{\tilde{y}^3 2y_d\tilde{y}^2 + y_d{}^2\tilde{y}}{l^2}\right)\right], \\
&= -kl\left[\tilde{y} - \frac{\tilde{y}^3}{l^2} + \frac{2y_d\tilde{y}^2}{l^2} - \frac{y_d{}^2\tilde{y}}{l^2}\right], \\
&= -k\left[l\tilde{y} - \frac{\tilde{y}^3}{l} + \frac{2y_d\tilde{y}^2}{l} - \frac{y_d{}^2\tilde{y}}{l}\right], \\
&= -k\left[-\frac{\tilde{y}^3}{l} + \frac{2y_d\tilde{y}^2}{l} + \left(l - \frac{y_d{}^2}{l}\right)\tilde{y}\right], \\
&= k\left[\frac{\tilde{y}^3}{l} - \frac{2y_d\tilde{y}^2}{l} - \left(l - \frac{y_d{}^2}{l}\right)\tilde{y}\right].
\end{aligned} \tag{26}$$

Table 1: Plant and regulator parameter values

Parameter	Value	Units
l	1	m
y_d	1/2	m
k	1	rad/m s
ε	10^{-3}	N/A

The qualitative shape of function $\phi(\tilde{y}, y_d)$ in Eq. 26 is depicted in Fig. 4. It vanishes at the equilibrium $y = 0$ and its borders.

The derivative of Eq. 26, depicted in Fig. 5, yields:

$$\frac{d}{d\tilde{y}}\varphi(\tilde{y}, y_d) = k\left[\frac{3\tilde{y}^2}{l} - \frac{4y_d\tilde{y}}{l} - \left(l - \frac{y_d{}^2}{l}\right)\right], \tag{27}$$

which vanishes at:

$$\beta_1 \triangleq \frac{4y_d - 2\sqrt{3l^2 + yd^2}}{6}, \tag{28}$$

$$\beta_2 \triangleq \frac{4y_d - 2\sqrt{3l^2 + yd^2}}{6}, \tag{29}$$

Both β_1 and β_2 correspond to critical points of function ϕ. They are illustrated in the plot of Fig. 4.

Evaluating Eq. 27 at the equilibrium $y = 0$ yields:

$$\frac{d}{d\tilde{y}}\varphi(0, y_d) = -l^2 + y_d^2 < 0.$$

Therefore, by invoking the so-called Lyapunov's indirect method (Khalil, 2002, Theorem 4.7, pg. 139), one has formally proven that the equilibrium y = 0 of nonlinear system Eq. 22 is an asymptotically stable one (for alternative analysis tools see also the book of (Wiggings, 2003), Theorem 1.2.5, pg. 11, or the Linearization Theorem in (Arrowsmith and Place, 1992), Theorem 3.3.1, pg. 77.

To study the asymptotic behavior of the Cartesian position error y, this study also shall invoke e

Lyapunov base arguments stated as Corollary 1 previously in this study. To this end, let us consider the following globally positive definite and radially unbounded functions:

$$z(\tilde{y}) = \frac{1}{2}\tilde{y}^2. \tag{31}$$

Following the notation of Corollary 1, Eq. 31 satisfies Eq. 3 where:

$$w_1(|\tilde{y}|) = w_2(|\tilde{y}|) = \frac{1}{2}|\tilde{y}|^2.$$

The time derivative of Eq. 31 along the closed-loop Eq. 22 yields:

$$\begin{aligned}\dot{z}(\tilde{y}) &= \tilde{y}\dot{\tilde{y}},\\ &= -kl\left[1-\left(\frac{\tilde{y}-y_d}{l}\right)^2\right]\tilde{y}^2,\\ &= -w_3(\tilde{y})\end{aligned} \tag{32}$$

where:

$$w_3(\tilde{y}) \triangleq kl\left[1-\left(\frac{\tilde{y}-y_d}{l}\right)\right]\tilde{y}^2 \tag{33}$$

is sketched in Fig. 6. This is a positive definite function in D already defined in Eq. 23.

The time derivative z˙(y) in Eq. 32 is a negative definite function in the region D, Fig. 7.

Concerning Corollary 1, this study proposes the following formula to choose constants *r* and *c* (Fig. 8):

$$r \triangleq \frac{\left|-l+y_d\right|}{1+\varepsilon}, \tag{34}$$

and:

$$c \triangleq \frac{1}{2}r^2 = \frac{1}{2}\left(\frac{\left|-l+y_d\right|}{1+\varepsilon}\right),$$

for any $\varepsilon \in (0, \infty)$.

According to numerical values in Table 1 one gets: $r = 0.4995$ and $c = 0.1248$.

Set B_r defined in Eq. 5 becomes:

$$\begin{aligned}B_r &= \{\|x\| \le r\},\\ &= \{|\tilde{y}| \le r\},\end{aligned}$$

Such a set B_r is strictly inscribed into domain D, i.e., $B_r \subset D$, in other words, the condition in Eq. 5 is fulfilled.

Thus, invoking Corollary 1, we conclude that the equilibrium $y = 0$ is locally asymptotically stable and an estimate of the domain of attraction in Eq. 7 is:

$$\begin{aligned}A &= \left\{\tilde{y} \in B_r \,\middle|\, w_2(\tilde{y}) \le c\right\},\\ &= \left\{\tilde{y} \in B_r \,\middle|\, \frac{1}{2}|\tilde{y}|^2 \le c\right\},\\ &= \left\{\tilde{y} \in B_r \,\middle|\, |\tilde{y}| \le \sqrt{2c}\right\},\\ &= \left\{\tilde{y} \in B_r \,\middle|\, |\tilde{y}| \le r\right\}.\end{aligned} \tag{35}$$

Given the values in Table 1 one gets:

$$A = \left\{\tilde{y} \in B_r \,\middle|\, |\tilde{y}| \le 0.4995\right\}.$$

One summarizes the main stability result in the following.

Proposition 2. Asymptotic Stability and Estimate of the Domain of Attraction

Consider the closed-loop system modeled by the nonlinear ordinary differential Eq. 25. Then, the trivial equilibrium $y = 0$ is asymptotically stable and an estimate of the domain of attraction is A in Eq. 35 with r in Eq. 34, i.e.:

$$A = \left\{\tilde{y} \in \mathbb{R} \,\middle|\, |\tilde{y}| \le \frac{1}{2}\left(\frac{\left|-l+y_d\right|}{1+\varepsilon}\right)^2\right\},$$

for any $0 < \varepsilon < \infty$.

Numerical Simulations

Numerical simulations of the nonlinear closed-loop differential Eq. 22 have been carried out utilizing the MATLAB® software calling upon the ODE45 numerical engine under configuration: Variable-step integration and relative tolerance 1e-3. Numerical values of parameters involved in Eq. 22 are listed in Table 1.

Per the numerical values of Table 1, the important sets are summarized in Table 2. Among them is the estimated domain of attraction:

$$A = \{|\tilde{y}| \le 0.4995\}.$$

Figure 9 depicts the resulting trajectories $y(t)$ for a set of eight initial conditions arranged in the form:

$$\begin{aligned}&\tilde{y}(0) \in \{-0.45, -0.25, -0.1\} \in A.\\ &\tilde{y}(0) \in \{0.50, 1.0, 1.45\} \in D - A.\\ &\tilde{y}(0) \in \{-0.5, 1.5\} \notin D.\end{aligned}$$

As one expected, initial condition inside the estimate of the domain of attraction A tend to the equilibrium y = 0. Also, for initial conditions starting in D-A (outside the estimate of attraction A), they converge to the equilibrium $y = 0$. This unexpected result shows that A is a conservative estimate of the domain of attraction.

Initial conditions: $y(0) = -0.5$ and $y(0) = 1.5$ correspond to spurious equilibria beyond the domain of definition D of the system.

Table 2: Sets involved in Corollary 1

Set	Interval of y	Numerical interval e
D	$-l+y_d < \tilde{y} < l+y_d$	$-0.50 < \tilde{y} < 1.50$
B_r	$\lvert\tilde{y}\rvert \le r = \frac{\lvert -l+y_d\rvert}{1+\varepsilon}$	$\lvert\tilde{y}\rvert \le 0.4995$
A	$\lvert\tilde{y}\rvert \le r = \frac{\lvert -l+y_d\rvert}{1+\varepsilon}$	$\lvert\tilde{y}\rvert \le 0.4995$

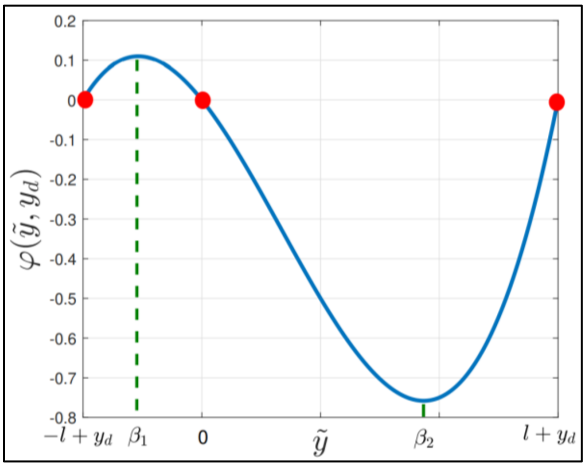

Fig. 4: Plot of function φ in Eq. 26, zeroes at small red full circles

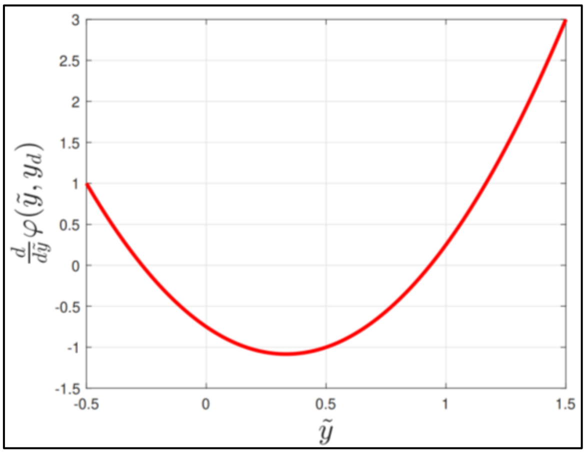

Fig. 5: Derivative of function φ within the valid interval $\tilde{y} \in (-0.5, 1.5) = D$

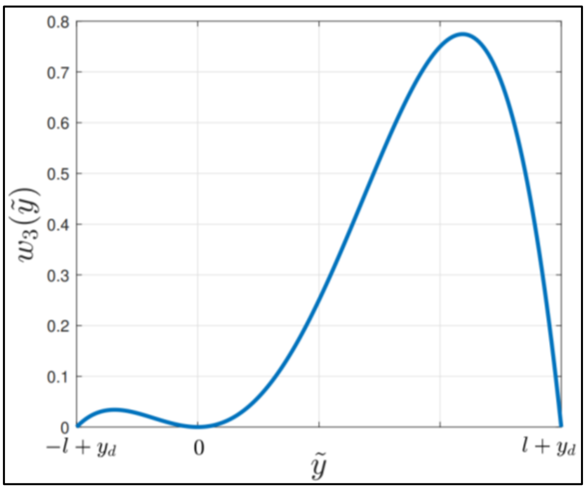

Fig. 6: Plot of $w_3\ (\tilde{y})$ in Eq. 33

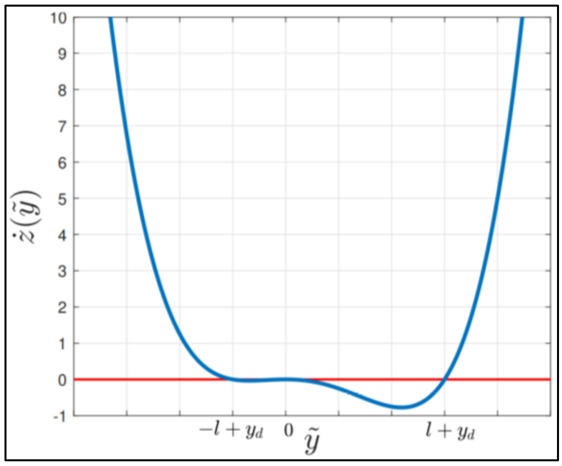

Fig. 7: Plot of $\dot{z}(\tilde{y}) = -\ w(\tilde{y})$ in Eq. 32. Function $\dot{z}(\tilde{y})$ is a locally negative definite function in the domain D

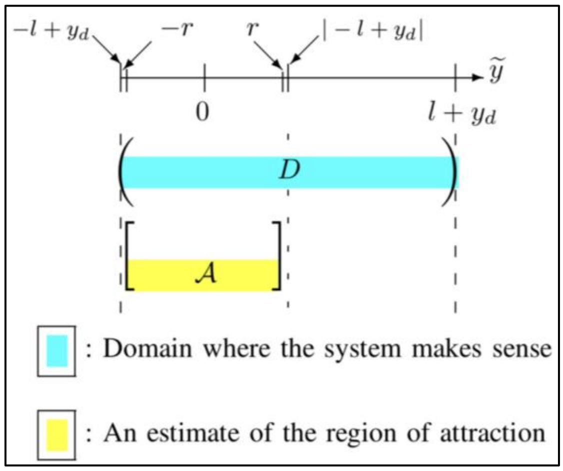

Fig. 8: Sets; D: System domain of definition, and A an estimate of the domain of attraction

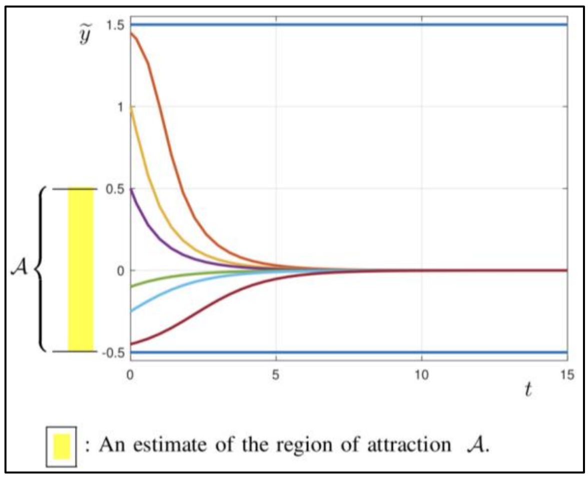

Fig. 9: Trajectories $\tilde{y}$ (t) for 8 initial conditions: $\tilde{y}$ (0) ∈ {−0.5, −0.45, −0.25, −0.1, 0.5, 1.0, 1.45, 1.5}

Conclusion

A didactic mechatronic device composed of an ideal position servo actuator powering a rigid pendulum rod endowed with an angular position sensor for feedback purposes has been modeled. This model is at the origin of an original nonlinear Cartesian regulator design whose advantage is that it avoids the annoying inverse of the kinematic mapping. An additional pro is that angular speed sensing is not needed by the proposed controller. The closed-loop system stability and an estimate of the basin of attraction have been established by invoking Lyapunov analysis tools. Numerical simulations illustrate the system's performance.

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Author's Contributions

Gabriela Zepeda: Controller designed, estimate domain of attraction, numerical simulations, manuscript writing, and review.

Rafael Kelly: Conceptualization, control problem formulation, stability analysis, manuscript writing.

Carmen Monroy: Conceptualization, figures elaboration, theory review.

Ethics

This article is original and contains unpublished material. The corresponding author confirms that all the other authors have read and approved the manuscript and no ethical issues are involved.

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