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Stochastic Model for Variable Amplitude Fatigue Induced Delamination Growth in Graphite/Epoxy Laminates

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Abstract: A stochastic model is used to predict the growth of delamination in graphite/epoxy laminates subjected to variable amplitude cyclic loading. The advantage of this model is that both the mean and variance associated with the growth of delamination are predicted. Understanding and predicting the variability associated with the delamination growth process is essential to the estimation of the reliability of composite structures. The empirical nature of the model has been minimized through the introduction of fracture mechanics parameters into the stochastic model. This has been accomplished by assuming that the mean of the stochastic model is represented by a fracture mechanics power law based on strain energy release rate. The applicability of the model is demonstrated through an experimental data generated on three laminate geometries. The results of the experimental evaluation illustrate the ability of the model to predict both the mean and variance of the delamination growth process in composite laminates subjected to cyclic loading.

Keywords: Composite Materials, Fatigue, Delamination, Experimental, Stochastic Model

Introduction

Laminated fiber reinforced materials are used in many commercial and military structures because of their superior specific properties and the ability to tailor the structural properties for a given application. However, the complex anisotropic and heterogeneous nature of these materials has hindered their optimization and broad acceptance. One of the most profound examples of this complexity is the accumulation and effect of the damage on the integrity of composite materials subjected to cyclic loading. In traditional homogenous materials, failure results from the propagation of a single crack. In composite materials cracks (both inter-laminar matrix cracking, intra-laminar matrix cracking and fiber breaks) form throughout the material during its life cycle and failure results from the accumulation of these various types of damage. This process is dependent on fiber architecture, materials used and structural geometry. A thorough understanding of damage accumulation is critical to structural assessment.

The approaches that have been traditionally taken to address the structural assessment of composite materials subjected to cyclic loading are summarized and reviewed by (Degrieck and Van Peagegem, 2011; Garnich and Akula, 2009). These approaches include empirical theories, residual strength degradation theories, stiffness degradation theories and damage mechanics theories. Empirical, residual strength and stiffness degradation approaches do not account for the underlying damage accumulation phenomenon that are inherent to the degradation of composite structures subjected to cyclic loading. As a result, their predictions are typically only applicable to a specific material and laminate combination. The damage mechanics approach has the potential of predicting damage accumulation and residual properties from basic material properties without the need for specific laminate testing because the underlying damage phenomenon are intrinsic to the damage mechanics approach. Although the fundamental understanding needed to make this approach successful has yet to be fully developed.

Cyclic loading of a composite structure will result in fiber, interlaminar and intralaminar damage accumulation. Investigators have explored the use of the fracture mechanic's principal of strain energy release rate (Paris and Sih, 1965) in modeling interlaminar (O'Brien, 1982; Wang *et al*., 1985; O'Brien *et al*., 1993; O'Brien, 1993) and intralaminar (Wang, 1984; Suresh and Wang, 1993; Sriram and Armanios, 1993) cracking. However, few have addressed the high degree of variability that is associated with the damage growth

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processes. An understanding and accurate assessment of the variability is critical to reliability evaluations.

The first attempts to address the probabilistic nature of the composite damage accumulation process were based on the Weibull (1951) extreme value type distribution. Harlow and Phoenix (1978a; 1978b; 1979) included fiber failure and Ramani and Williams (1977) and Talreja (1987) included fatigue life models. The Weibull approach is limited because the probability distribution has not been related to the damage process, it is not possible to separate and account for the major sources of variability in material response, the correlation characteristics of test data are not accounted for and the nature of physical process are not reflected. Further, there are only two limiting states of the material in Weibull distribution based fatigue models: Satisfactory (no damage) and failure. A rigorous stochastic approach can take into account all sources of variability in material response and lead to a reliable prediction of damage growth. Wang *et al*. (1984) presented a stochastic model for intralaminar crack growth and Bucinell (1998) presented a stochastic model for interlaminar damage growth.

In the remainder of this paper the fracture mechanics based stochastic delamination growth model developed by Bucinell (1998) is further developed for variable amplitude intralaminar damage growth and compared to experimental data. First the experimental program designed to generate data for the model is presented. Then the stochastic model development is summarized. This is followed by a discussion of the application of the model to the growth of delamination under constant amplitude fatigue loading for three laminates. Following this, the parameters of the model are scrutinized and the arguments are made in favor of categorizing some of the parameters as material properties.

Materials and Methods

An experimental study was conducted to demonstrate the use of a fracture mechanics based model to predict the growth of delamination under variable amplitude fatigue loading. The validation of the model required several phases. First the basic ply material properties, the laminate stacking sequence and the maximum fatigue load amplitude on the mean growth of delamination are experimentally determined. The secondly the scatter in the growth of delamination is quantitatively characterized using constant amplitude loading for later use in the cumulative model. The results of the experimental investigation are used in the development and validate the cumulative damage model presented later in this study. Both static and cyclic fatigue tests were conducted to provide basic laminate property data for the model and to provide data for damage accumulation comparisons.

Material Description

The basic material system used to fabricate the laminates was an AS4 fiber/3501-6 resin unidirectional graphite/epoxy prepreg tape. Several panels were fabricated in the $[\pm 25/90]_s$ laminate geometry for this investigation. These panels were then autoclaved cured to the manufacture's specifications. After cure, fiber-glass/epoxy end-tabs were attached to the panels using a cement. The panels were then cut into straight tensile coupons 300 mm (12 in) long by 19 mm (0.75 in) wide.

Experimental Procedures

To achieve the experimental objective, three sets of test were conducted. The first set of tests determined the elastic properties and the ultimate tensile strengths of the various laminates. The second set of tests established the delamination threshold, the load-delamination growth relationship and location of the delamination interface under a static loading. This data was used to characterize static delamination growth and to define the load levels for the constant amplitude fatigue tests. The third and final set of tests established the load-time-delamination growth relationship and the location of the delamination interface under constant and variable amplitude fatigue loading.

Uniaxial tensile static and fatigue tests were performed on a closed-loop servo-hydraulic Instron machine. All static tests were performed in displacement control and the cross-head speed was set to 0.25 mm (0.01 in.) per minute. For the fatigue tests, the load frame was placed in load control and a sinusoidal wave form at a frequency of 6 Hz was used. The minimum to maximum load amplitude ratio was set to R = 0.1. Matrix damage in the test coupons was monitored non-destructively using an X-radiographic system (Hewlett Packard FAXTRON 800).

In the first set of tests, bi-axial strain gage rosettes were mounted on each coupon for the purpose of characterizing the laminate properties. The longitudinal laminate modulus, E_x, Poisson ratio, ν_{xy} and the ultimate laminate strength were experimentally determined. The mean properties from these tests are summarized in Table 1.

The second set of tests used incremental static loading to monitor the location and growth of delamination in the test specimens. Each coupon in this set of tests was loaded to predetermined load increments and then the load was relieved. The specimens were removed from the test fixtures and inspected for free edge delamination using X-radiography and then reloaded to a higher load level. The unloading and reloading of the specimens to higher predetermined levels was continued until the specimen failed. The free-edge interface where the delamination occurred was determined optically using a 10× magnifying lens during the incremental loading. Several specimens were selected to be photomicrographed in order to record the

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location of the delamination interface. The experimentally observed location of the delamination interface for this laminate is summarized in Table 2 along with the static delamination onset stress and the breaking strength of the laminates. The ultimate strength of the laminates during the incremental static load testing (Table 2) compare well with the ultimate strength determined during the continuous static load testing (Table 1). This implies that the repeated loading and unloading of the incremental tests and the X-radiography enhancement solution did not affect the damage growth characteristics of these laminates under static loading.

The third set of tests were conducted under constant and variable amplitude fatigue loading The constant amplitude fatigue load level testing was conducted at three fatigue load levels. One load level was chosen near the static delamination onset load, another was chosen so that the coupon would fail around one million cycles and a third was chosen between the other two. The three load levels are summarized in Table 3.

During fatigue testing the cycling was halted at a series of predetermined cycles and the specimens were removed from the test fixture. Using the same x-radiography inspection procedure followed during the static testing, the growth of delamination was monitored. The specimens were then reloaded to a higher predetermined cyclic increment. This procedure was repeated until the specimen failed. The edge of the specimens were also examined for the location of the delamination interface. A summary of the load levels at which the constant amplitude fatigue tests were conducted along with the location of the delamination interface are found summarized in Table 3.

Table 1. Mean [±25/90]s laminate properties determined in the mechanical property characterization testing phase of study

Replicates	E_x GPa (Msi)	ν_{xy}	Ultimate stress MPa (ksi)
2	64.4 (9.34)	0.290	406. (59.0)

Table 2. Results of incremental static loading phase of the experimental program for the [±25/90]s laminate. The average stress for the onset of delamination, the ultimate stress of the laminate and the interface where the delamination formed are reporte

Replicates	Ultimate stress MPa (ksi)	Onset stress MPa (ksi)	Interface
2	338. (49.0)	400. (58.0)	-25/90

Table 3. Summary of the load levels at which [±25/90]s laminate specimens were tested during the cyclic loading phase of the experimental program

Replicates	Maximum fatigue load MPa (ksi)	Delamination interface
3	200. (29.0)	-25/90
3	228. (33.0)	-25/90
3	276 (40.0)	-25/90

Stochastic Delamination Growth Model

A stochastic process is a mathematical model of any dynamic process whose evolution with time is governed by some probabilistic law. The advantage of modeling fatigue delamination growth as a stochastic process is that the inherent variability in the growth of delamination can be predicted. The stochastic model by Bucinell (1998) was developed to predict the growth of delamination in composite laminates. This model assumes that the number of cycles needed for a delamination to grow to a given length is best described by a random variable. The parameters of the probability distribution used to describe this random variable are assumed to vary with the applied fatigue load level. By describing these parameters in terms of the applied fatigue load level, the amount of testing required to correlate the model can be significantly reduced.

Bucinell (1998) describes the stochastic model parameters in terms of the applied fatigue load level is based on the assumption that a deterministic fracture mechanics model can be equated to the mean expression of the stochastic model. Hence, the parameters in the stochastic model can be directly related to the applied fatigue load. This development is facilitated by discretizing both time and delamination size. A discretized time increment is referred to as a cycle. A cycle is defined as a repetitive period of cycling during which delamination can occur. In this development, each fatigue load reversal is considered a cycle. A discretized delamination size increment is referred to as a delamination state.

The following restrictions are imposed on the definitions of a cycle and a delamination state in the development of this model:

- The increment of delamination damage at the end of a cycle depends only on the delamination state present at the start of that cycle
- Delamination damage can only increase during a cycle
- No initial delamination or manufacturing flaws exist in the virgin laminate that are on the order of a delamination state. Thus, all delamination damage starts in the undamaged state
- Between cycles, only the fatigue load level can change. The load frequency, R-ratio, environmental conditions, etc., are all assumed to remain constant

Proceeding with these assumptions in mind, let the probability of the delamination advancing from the existing delamination state to the next during any cycle be assigned the value p. The probability of the delamination not advancing to the next delamination state is assigned the value q. Since it is assumed that the delamination can only increase by one delamination state during any cycle or remain in the

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same delamination state during that cycle, the following equality can be written:

$$p + q = 1 \tag{1}$$

The values of p and q remain constant as long as the fatigue load level remain constant. Changes in the fatigue load level will result in changes in p and q.

Bucinell (1998) development then goes on to show that the constant amplitude fatigue delamination growth process is described by the Negative Binomial distribution (Hastings and Peacock, 1975). Where the mean (E[•]) and variance (var[•]) are given by:

$$E[T_n] = n / p \tag{2}$$

$$\text{var}[T_n] = n(1-p)/p^2 = n\left(p^{-2} - p^{-1}\right) \tag{3}$$

In these expressions T_n is the random variable defined as the number of cycles needed for a delamination to progress from the initial delamination state, (0)th, to the (n)th delamination state.

At this point the values of parameters p and n can be estimated from constant amplitude fatigue delamination growth data using regression analysis. Since these parameters change with the fatigue load amplitude, they will have to be estimated for each load level under consideration unless a relationship between these parameters and the fatigue load level can be developed.

Experimental evidence (Wang *et al*., 1985; Ye, 1989; Dahlen and Springer, 1994) suggests that the fracture mechanics power law shown in Equation 4, written in terms of the strain energy release rate G and the critical strain energy release rate G_c, describes the mean growth behavior of delamination:

$$\frac{da}{dN} = \alpha\left[G(\sigma_m, a)/G_c\right]^{\rho} \tag{4}$$

Wang and Crossman (1980) showed that G can be expressed explicitly in terms of the applied load as:

$$G(\sigma_m, a) = C_e(a) \cdot t \cdot \sigma_m^2 / E^2, \tag{5}$$

Where:

σ_m = The applied load
E = The elastic modulus of the laminate in the direction of the load
$C_e(a)$ = A coefficient function which depends only on the delamination size a and t is the thickness of a ply

By assuming C_e is a constant (Bucinell, 1998), Equation 4 can be easily solved for the mean number of fatigue cycles required for the delamination to reach a given length, a:

$$N = (a/\alpha)\left[\left(C_e \cdot t \cdot \sigma_m^2\right)/\left(G_c \cdot E^2\right)\right]^{-\rho} \tag{6}$$

Equation 6 is a continuous representation of the mean delamination growth rate. Both Equations 2 and 6 describe the same event. Discretizing Equation 6 enables the parameters of the probabilistic model (Equation 2) to be written in terms of fracture mechanics parameters.

The discretization of Equation 6 starts by letting a_{ds} be the size of a delamination state and n be the number of delamination states to a fixed crack length $a = a_f$:

$$a_f = a_{ds} \cdot n \tag{7}$$

It follows that:

$$\begin{aligned} E[T_n] &= n/p = N \\ &= \left(\frac{a_{ds} \cdot n}{\alpha}\right) \cdot \left[\frac{\left(C_e \cdot t \cdot \sigma_m^2\right)}{\left(G_c \cdot E^2\right)}\right]^{-\rho} \\ &= \left(\frac{a_f}{\alpha}\right) \cdot \left[\frac{\left(C_e \cdot t \cdot \sigma_m^2\right)}{\left(G_c \cdot E^2\right)}\right]^{-\rho} \end{aligned} \tag{8}$$

With a substitution of Equation 6 and 7 into Equation 2, the probability of a delamination state advancing to the next state on a given cycle, in terms of fracture mechanics parameters, is given by:

$$p = (\alpha / a_{ds})\left[\left(C_e \cdot t \cdot \sigma^2\right)/\left(G_c \cdot E^2\right)\right]^{\rho} \tag{9}$$

A similar procedure is used to write the variance in terms of fracture mechanics parameters. The development starts with the variance of the constant amplitude fatigue delamination growth process, Equation 3. This development is based on the assumption that the values of p are on the order of 10^{-3} and smaller. With this assumption Equation 3 can be simplified to:

$$\text{var}[T_n] = T_{sd}^2 = n/p^2 \tag{10}$$

where, T_{sd} represents the standard deviation of the random variable T_n. Substituting Equations 7 and 9 into Equation 10 and placing stress related terms on the left-hand side of the equation yields:

$$a_{ds} \cdot \sigma^{-4 \cdot \rho} = T_{sd}^2 \left(\frac{\alpha^2}{a_{ds}}\right)\left[\frac{C_e \cdot t}{G_c \cdot E^2}\right]^{2 \cdot \rho} \tag{11}$$

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Load level is known to have a direct effect on mean fatigue delamination growth behavior. It is anticipated that load level will also have an effect on the variability associated with fatigue delamination growth behavior. Since the delamination state size a_{ds} is influenced by this variability, it can be concluded that a_{ds} is a function of load level. This conclusion is supported by data presented by (Wang and Crossman, 1980). The manner in which a_{ds} varies with the load level is assumed to be of the general form:

$$a_{ds} = A\sigma^{B} \tag{12}$$

Substituting Equations 7 and 12 into Equation 11 yields:

$$\sigma^{B-4\rho} = T_{sd}^{2} \cdot \left(\frac{\alpha^{2}}{A \cdot a_{f}}\right) \cdot \left(\frac{C_{e} \cdot t}{G_{c} \cdot E^{2}}\right)^{-2\rho} \tag{13}$$

The implication of this equation is that at a fixed delamination size, a_f, there is a relationship between the load level and the variance in the number of cycles to reach that delamination size, T_{sd}^{2}.

Taking the logarithm of both sides of Equation 13 and performing some simplification results in the following expression:

$$\log(\sigma) = B_{v1} \cdot \log(T_{sd}) + B_{v0} \tag{14}$$

where:

$$B_{V1} = \left(2/B - 4\rho\right) \tag{15}$$

and:

$$B_{v0} = \left(\frac{1}{B-4\rho}\right) \cdot \log\left\{\left(\frac{\alpha^{2}}{A \cdot a_{f}}\right)\left[\frac{C_{e} \cdot t}{G_{c} \cdot E^{2}}\right]^{2\rho}\right\} \tag{16}$$

Thus the variance of T_n can be written:

$$\begin{aligned} \mathrm{var}[T_n] &= T_{sd}^{2} \\ &= 10^{-2 \cdot B_{v0}/B_{v1}} \cdot \sigma^{2/B_{v1}} \end{aligned} \tag{17}$$

At this point the model parameters p and n, can be estimated directly from fracture mechanics parameters. Solving Equations 2 and 9 simultaneously yields:

$$p = \left[1 + \left(\mathrm{var}[T_n] / E[T_n]\right)\right]^{-1} \tag{18}$$

$$n = E[T_n]\left[1 + \left(\mathrm{var}[T_n] / E[T_n]\right)\right]^{-1} \tag{19}$$

where, the mean, $E[T_n]$ and variance, $var[T_n]$, are defined in terms of fracture mechanics parameters in Equations 8 and 17.

The stochastic delamination growth model has been completely developed for the case of constant amplitude fatigue loading. Experimental data at several fatigue load amplitudes is required to estimate the parameters of the model and then the model can be used to estimate the mean and variance in the growth of delamination at several other fatigue load levels. The following section discusses the correlation of the model with actual fatigue data.

Extending the Model to Variable Amplitude Loading

For the case of variable amplitude loading, the mean and variance of T_n are developed using the characteristic function described by Bucinell (1998):

$$h_T(u) = \prod_{j=1}^{n} p_j \cdot e^{iu} / 1 - q_j \cdot e^{iu} \tag{20}$$

The central moments of a random variable can be calculated from the characteristic function by:

$$E\left[T^{k}\right] = \frac{1}{i^{k}} \cdot \frac{d^{k}}{du^{k}} \cdot h_T(0) \tag{21}$$

where, $E[T^k]$ represents the (k)th central moment of T_n. The first central moment corresponds to the mean of the random variable. It follows from Equation 21 that the mean of T_n for the case of variable amplitude fatigue loading is:

$$\begin{aligned} E[T_n] &= 1/p_1 + 1/p_2 + \cdots + 1/p_n \\ &= E[X_1] + E[X_2] + \cdots + E[X_n] \end{aligned} \tag{22}$$

This equation implies that even if the X_js are not identically distributed, the mean of T_n is a summation of the means of the individual X_js.

The variance of T_n for the case of variable amplitude fatigue loading is defined in terms of central moments as:

$$\mathrm{var}[T_n] = E\left[T_n^{2}\right] - E\left[T_n\right]^{2} \tag{23}$$

Both terms on the right-hand side of this equation can be calculated using Equation 21. It follows that the variance of T_n can be written:

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$$\begin{aligned}\operatorname{var}[T_n] &= \frac{1-p_1}{p_1^2} + \frac{1-p_2}{p_2^2} \\ &\quad + \cdots + \frac{1-p_n}{p_n^2} \\ &= \operatorname{var}[X_1] + \operatorname{var}[X_2] \\ &\quad + \cdots + \operatorname{var}[X_n]\end{aligned} \tag{24}$$

The variance of T_n is simply the summation of the variances of the X_js.

To this point, the model development has shown that the variable amplitude fatigue process can be represented by a summation of constant amplitude fatigue processes. Thus, the parameters of the model can be estimated through a series of constant amplitude fatigue experiments. From these experiments, the mean and variance of the number of fatigue cycles to a fixed crack length are calculated at each fatigue load level being considered. From this information, Equations 2 and 3 are used to estimate the p's and n's that correspond to each of the fatigue load levels. Then Equations 23 and 24 can be used to predict the mean and variance of the prescribed variable amplitude fatigue load history.

The cumulative damage description using equations 23 and 24 imply that the change in fatigue load level occurs only at damage state increments. Typically, in cumulative damage tests, the load level is changed at a prescribed number of fatigue cycles. These usually do not coincide with the damage state increments. Thus, the process describing the number of damage states a delamination has progressed through in t duty cycles needs to be considered. Let the random variable D_t describe this process. Expressions for the distribution function, mean and variance of this random variable now need to be developed.

Consider the event that on the (t)th duty cycle the delamination has progressed n or less damage states. In terms of the random variable D_t this event is written:

$$D_t \le n. \tag{25}$$

Figure 1 illustrates this event. For this event to occur, the number of duty cycles to the (n+1)th damage state must occur after the (t)th duty cycle. This is written in terms of the random variable T as:

$$T_{n+1} > t. \tag{26}$$

Thus, Equations 25 and 26 are equivalent events. In terms of probabilities, the equivalence of these events can be written:

$$\Pr[D_t \le n] = \Pr[T_{n+1} > t] \tag{27}$$

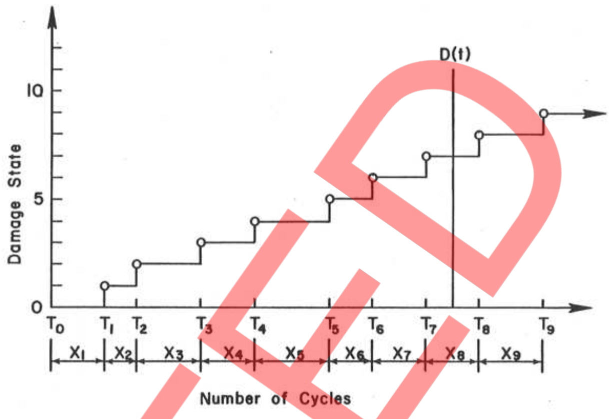

Fig. 1. Illustration of the relationship between the random variables in the stochastic model where the on the (t)th duty cycle the delamination has progressed n or less damage states

It follows:

$$\Pr[D_t > n] = \Pr[T_{n+1} \le t] \tag{28}$$

The right hand side of Equation 28 is the cumulative distribution function of the summation of duty cycles spent in each of the individual damage states. Using the notation presented by Bucinell (1998), Equation 28 can be written:

$$\Pr[D_t > n] = F_{n+1}(t), \quad t, n = 1, 2, \ldots \tag{29}$$

This equation enables the probability density function of D_t, $f_t(n)$, to be expressed in terms of the random variable T_n:

$$\begin{aligned}\Pr[D_t = n] &= \Pr[D_t > n-1] \\ &\quad - \Pr[D_t > n] \\ &= F_n(t) - F_{n+1}(t) \\ &= f_t(n)\end{aligned} \tag{30}$$

Thus the cumulative distribution function of D_t, is defined as:

$$\Pr[D_t \le n] = F_t(n) = \sum_{u=1}^{\infty} f_t(u) \tag{31}$$

The mean and variance of D_t are defined respectively as:

$$E[D_t] = \sum_{j=1}^{\infty} j \cdot f_t(u) \tag{32}$$

and:

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$$\mathrm{var}\left[D_t\right] = \sum_{j=1}^{\infty} j^2 \cdot f_t(j) - E\left[D_t\right]^2 \quad (33)$$

Since the D_ts are considered to be independent, it follows that:

$$E\left[D_t\right] = E\left[D_{t1}\right] + E\left[D_{t2}\right] + \cdots + E\left[D_{tn}\right] \quad (34)$$

and:

$$\mathrm{var}\left[D_t\right] = \mathrm{var}\left[D_{t1}\right] + \mathrm{var}\left[D_{t2}\right] + \cdots + \mathrm{var}\left[D_{tn}\right] \quad (35)$$

where, the load levels of the D_{tj}s are different.

Application of Model to Experimental Data

The stochastic model developed above can now be compared to experimental data. This process begins by using experimental data to estimate the parameters of the model (α, ρ, B_{V0} and B_{V1}). This requires delamination growth versus fatigue cycle data at a minimum of three fatigue load levels for each of the specimen geometries (the load levels are summarized in Table 3). Additionally, the energy released as the delamination opens is estimated using a finite element simulation presented by Bucinell (1998).

The parameters α and ρ are estimated using the delamination length versus number of cycles data for various fatigue load levels. A first order linear regression equation is formed by taking the logarithm of both sides of Equation 6 and arranging the resulting equation in the form:

$$Y = B_{m1} \cdot X + B_{m0} \quad (36)$$

Where:

$$Y = \log(a_i / N_i),$$
$$X = \log \sigma,$$
$$B_{m1} = 2 \cdot \rho, \text{ and}$$
$$B_{m0} = \log \alpha \cdot \left[\frac{C_e \cdot t}{E^2 \cdot G_c} \right]^{\rho}$$

The parameters α and ρ are estimated directly from the regression parameters B_{m1} and B_{m0}:

$$\hat{\rho} = \hat{B}_{m1} / 2$$
$$\hat{\alpha} = 10^{\hat{B}_{m0}} \cdot \left[\frac{C_e \cdot t}{E^2 \cdot G_c} \right]^{-\hat{B}_{m1}/2} \quad (37)$$

where, the "^" indicates an estimated parameter. The parameter C_e is calculated using a quasi-three-dimensional finite element model that employs the crack closure method and the moduli estimated from the previously discussed static experimental evaluation performed by Bucinell (1998). For the $[\pm 25/90]_s$ laminate the required fracture mechanics parameters are summarized in Table 4.

Figure 2 illustrates the regression used to estimate the parameters in Equation 37. The resulting estimates of α and ρ and their 95% confidence limits for the [±25/90]s laminate are summarized in Table 5.

The stochastic model developed in the preceding section requires the estimation of the parameters B_{V0} and B_{V1} in Equation 14. This estimation requires that data for the variance in the time it takes the delamination to reach a width a_f be plotted in log T_{sd} versus lot σ space. Equation 14 suggests that a first order linear regression is appropriate for the estimation of B_{V0} and B_{V1}; however, because of the limited amount of this type of data a Least Squares Analysis of Variance cannot be performed. Instead the Jackknifing technique presented by Miller (1974) was used. Figure 3 illustrates the Jackknifed estimation of B_{V0} and $B_{V1}$for $[\pm 25/90]_s$ laminate tested in this study. Table 6 summarizes the mean and variance for B_{V0} and B_{V1}.

Once B_{V0} and B_{V1} are estimated, the parameters A and B in Equation 12 can be calculated as follows:

$$B = \frac{2}{\hat{B}_{v1}} + 4\hat{\rho}$$
$$A = \frac{\hat{\alpha}^2}{a_f} \cdot \left[\frac{C_e \cdot t}{G_c \cdot E^2} \right]^{2 \cdot \hat{\rho}} 10^{-\hat{B}_{v0} \cdot (\hat{B} - 4 \cdot \hat{\rho})} \quad (38)$$

A summary of the regression parameters A and B calculated from Equation 38 for the $[\pm 25/90]_s$ laminate are found in Table 6. The calculations of A and B are based on the mean values of B_{V0} and B_{V1}.

Now that all of the parameters of the model have been estimated, the model can be used to predict the behavior of the growth of delamination for the load levels under consideration. A summary of the model parameters $E[T_n]$, T_{sd}, p and n, using the estimated parameters in Table 5 and 6, for the load levels under consideration are found in Table 7.

Two sets of variable amplitude fatigue tests were performed to evaluate the model. Periodically during the loading the cycling was stopped so the growth of damage could be measured using radiography. First, two $[\pm 25/90]_s$ laminates were subjected to 276MPa (40 ksi) for 1000 cycles and then 345 MPa (50 ksi) for an additional 100 cycles. Figure 4 shows the model prediction and experimental data for the last phase of the variable amplitude load spectrum. Second, two $[\pm 25/90]_s$ laminates were subjected to 345 MPa (50 ksi) for an additional 10 cycles and then 207 MPa (30 ksi) for an additional 250 000 cycles. Figure 5 shows the model prediction and experimental data for the last phase of the variable amplitude load spectrum.

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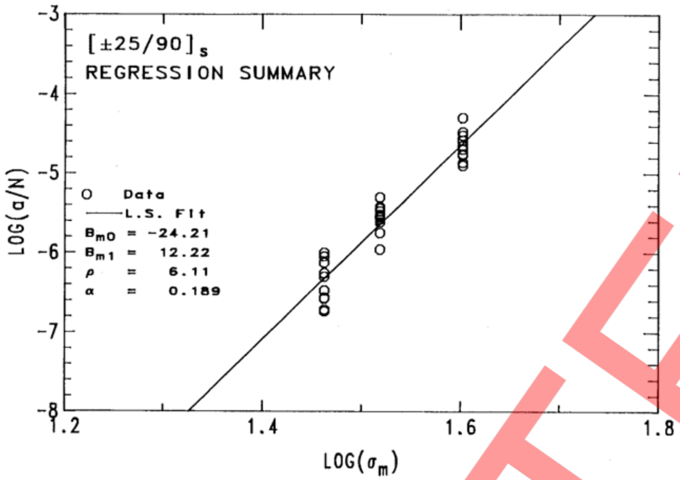

Fig. 2. Experimental data used to perform the linear regression to estimate the parameters B_{m1} and B_{m0} in Equation 37 and α and ρ

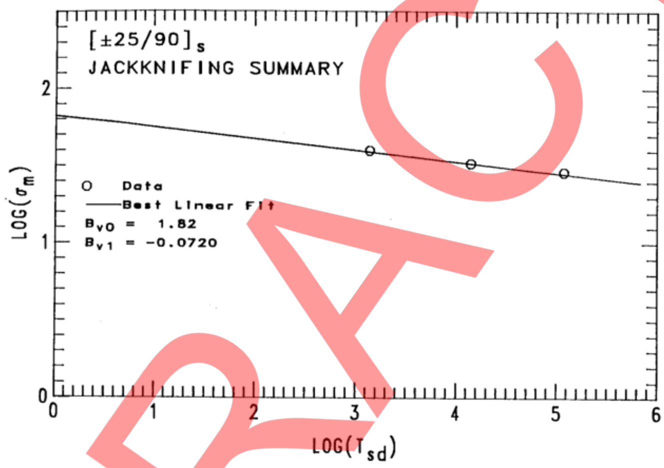

Fig. 3. Illustration of the Jackknifing technique used to estimation of the parameters B_{v0} and B_{v1} in Equation 14

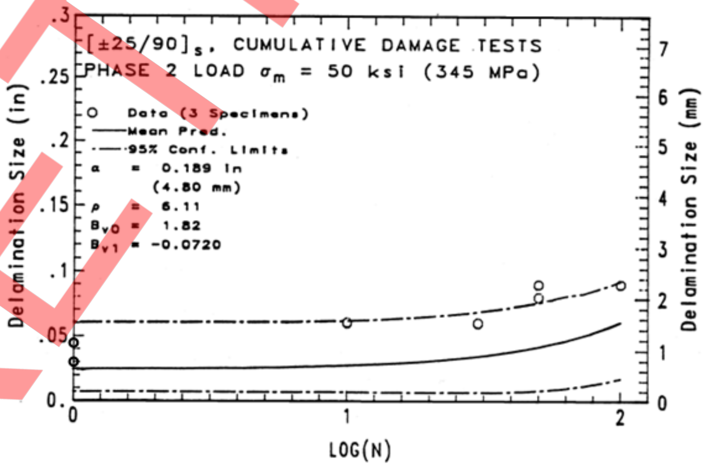

Fig. 4. Model prediction for a $[\pm25/90]_s$ laminate subjected to a first phase of a 276 MPa (40 ksi) load for 1000 cycles and then a second phase of 345 MPa (50 ksi) load for an additional 100 cycles, compared to experimental data for two specimens. Only the last phase of delamination growth is shown

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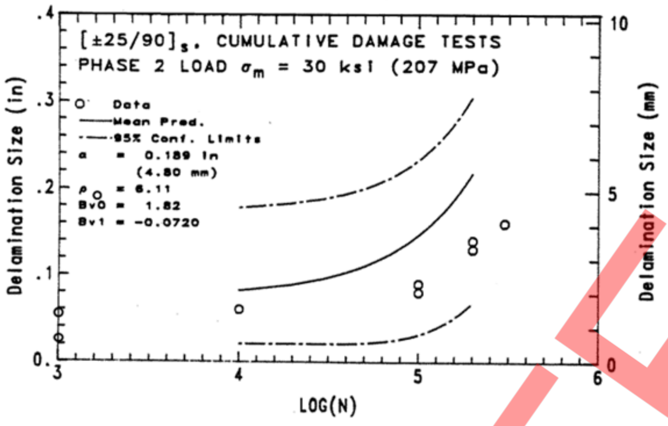

Fig. 5. Model prediction for a $[\pm25/90]_s$ laminate subjected to a first phase of a 345 MPa (50 ksi) load for 10 cycles and then a second phase of 207 MPa (30 ksi) load for an additional 250 000 cycles, compared to experimental data for two specimens. Only the last phase of delamination growth is shown

Table 4. The $[\pm25/90]_s$ laminate specific constants used in the calculation of $\hat{\alpha}$ and $\hat{\rho}$ for a delamination in the -25/90 interface

E_s [GPa] ([Msi])	C_e [GN/m^2] ([Mlb/in^2])	G_c [J/m^2] [(lb/in)]	t [mm] ([in])
64.4 (9.34)	20.1 (3.14)	227 (1.30)	0.13 (0.0052)

Table 5. First order linear regression estimates of fracture mechanics parameters α and ρ from $[\pm25/90]_s$ laminate experimental data generated using Equations 34 and 35

Mean ρ	95% Conf. Range	Mean α Mm (in)	95% Conf. Range Mm (in)
6.115	5.481-6.749	4.81 (0.1892)	1.51-15.3 (0.0593-0.6042)

Table 6. A summary of the parameters B_{v0}, B_{v1}, A and B found in Equations 12 and 14 for the $[\pm25/90]_s$ laminate

B_{v0}		B_{v1}			
Mean	Std Dev	Mean	Std Dev	A ($\times10^3$)	B
1.820	0.047	-.0720	0.0114	1.93	-3.338

Table 7. A summary of parameters T_{sd}, p and n for the $[\pm25/90]_s$ laminate in this study

Load level [Mpa] ([ksi])	E[T_n] Cycles	T_{sd} Cycles	p	n
200 (29.0)	1.59×10^{5}	9.28×10^{4}	1.85×10^{-5}	3
207 (30.0)	1.05×10^{5}	5.80×10^{4}	3.12×10^{-5}	3
228 (33.0)	3.28×10^{4}	1.54×10^{4}	1.38×10^{-4}	5
276 (40.0)	3.13×10^{3}	1.07×10^{3}	2.73×10^{-3}	9
345 (50.0)	2.05×10^{3}	4.81×10^{1}	8.86×10^{-2}	18

Discussion

The intent of the stochastic delamination growth model is to predict the growth of delamination in laminated composite materials subjected to variable amplitude fatigue loading. The model predictions include the mean growth of delamination and the variability about the mean growth of delamination. The model integrates fracture mechanics parameters so the model parameters can be estimated using constant amplitude fatigue data. The estimated parameters are then used to predict delamination growth under variable amplitude loading at load levels other than those used to predict the model parameters. The predictions of the mean and variability of the delamination growth shown in Figures 4 and 5 compare well with the delamination growth data.

The specimens examined in the experimental program did not contain any type of foreign substance in the layer interface that forced the delamination growth. The site of delamination onset and growth was allowed to occur naturally. The site of delamination in the $[\pm25/90]_s$ laminate was observed in the -25/90 interface for both static and fatigue loading (Table 2 and 3). At this interface, the finite element prediction of the energy release rate predicts a mixture of mode I and II fracture components (Bucinell, 1998). It is interesting to note that the total energy release in the -25/90 interface was lower than the midplane and the +25/-25 interface of this laminate. The midplane and the +25/-25 interface had nearly all mode I fracture components.

A few observations can be made about the location of delamination onset and propagation in composite laminates subjected to static and fatigue loading. First, in order for a delamination to form at an interface the normal opening edge stresses have to be very high and tensile. The finite element analysis performed on the $[\pm25/90]_s$ laminate shows that the interface where the delamination did form had singular normal opening stresses at the free edge (Bucinell, 1998). If a high tensile state of stress exists at an interface and the loading is static, the growth of delamination will be dominated by mode I fracture. If a high interlaminar tensile state of stress exists at an interface and the loading is cyclic, it appears that the interface with the maximum mode II component will control the growth of delamination.

The stochastic model developed provides a methodology for the prediction of the mean delamination growth rate in laminated composites subjected to constant amplitude cyclic loading and an estimate of the variability associated with the growth of the delamination. The model is developed without any predisposition to a probability distribution. Additionally, fracture mechanics parameters have been introduced into the model in order to minimize the amount of data required to correlate the model and to maximize the predictive capability of the model.

The stochastic model developed here implies that the delamination grows uniformly along the length, toward the center of the specimen. The crack length being predicted is actually the width of the delamination measured from the edge of the specimen. Observations of the delamination growth for the laminates under consideration in this investigation show that the delamination does occur along the entire length of the laminate, however, it is not uniform. O'Brien *et al*. (1993) also made a similar observation about the growth of edge delamination and went on to show that the error associated with assuming a uniform delamination front was negligible when compared to a detailed crack front model.

The stochastic model was developed around the assumption that a delamination may or may not advance during any fatigue cycle. These two events are assigned the probabilities p and q. The number of cycles to a given delamination size is found to be Negative Binomially distributed. This distribution captures the true discrete progressive nature of delamination growth during cyclic loading. It is not surprising that this distribution is not the same as the distribution associated with the accumulation of damage in fibers and the strength of laminates. The damage associated with these phenomenon occur at multiple sites in the composite and then coalesce into a critical damage state. This type of process has been shown by Harlow and Phoenix (1978a; 1978b; 1979) to follow a Weibull distribution.

The mean and variance predictions of the stochastic delamination growth model are compared to experimental data for each of the laminates in Fig. 4 and 5. All of the data obtained is found to lie within the 95% confidence limits predicted by the model. By changing the load parameter, σ, in Equations 8 and 17 the model can be used to predict the mean growth of delamination and the variability associated with this growth at load levels other than those used to correlate the model. This prediction is expected to be most accurate within the range of the load levels used to correlate the model. There is always a danger in extrapolating models outside the range of correlation.

The forms of the equations used to estimate the mean and variance parameters for the delamination growth model from experimental data, Equations 14 and 20, are both linear. Thus, in designing experiments to correlate the model for a given laminate, data should be generated at three load levels. Two of the load levels should correspond to the upper and lower bounds of the loading under consideration. The third load level should correspond to a load level mid-way between the upper and lower bound. The majority of the experimental data should be concentrated at the upper and lower bounds. This strategy will provide the most accurate estimates of the variability about the mean prediction of the model. This strategy was not employed during this experimental program. When this study was being carried out the form of the estimation curves were not known; therefore data was arranged such that a relationship as high as a second-order polynomial could be evaluated.

The scant amount of data used in the estimation of the parameters B_{V0} and B_{V1} can be traced to the meaning of the data points. Each of the data points represent the variance in the time it takes a delamination to grow to a certain length at a given load level. Thus the generation of a single data point in this figure requires the use of all the data at a given load level. The number of data available for the estimation of B_{V0} and B_{V1}, at each load level, can only be increased if several batches of material are used. If this is done, several specimens from each batch of material can be tested at each load level. The variability of the delamination growth to a specified length, at a given load level, for each batch provides an independent data point that can be used in the estimation of B_{V0} and B_{V1}. In doing so, a more accurate estimate of the mean and variance of B_{V0} and B_{V1} will be estimated.

Equations 15 and 16 show B_{V0} and B_{V1} as functions of mostly fracture mechanics parameters. The non-fracture mechanics parameters in these equations are A and B. These parameters were introduced to mathematically model the effect of load level on the damage state size a_{ds}. The nature of A and B is summarized in Table 6. From Equation 12, the negative sense of the parameter B indicates that as the load level increases the size of a delamination state decrease. The implication of this is seen in Equation 12 where a decrease in delamination state corresponds to an increase in the probability of the delamination advancing during any fatigue cycle. If further investigation reveal the true nature of parameters A and B, it could be possible to reduce the amount of data required to correlate the model. This could possibly lead to the use of data obtained with one material in a known laminate configuration predicting the mean and variance of the growth of delamination of the same material in a different laminate configuration. Further investigation into the nature of these parameters is required.

Summary and Conclusion

A delamination growth model for composite laminates subjected to variable amplitude fatigue loading has been developed. The parameters of the model have been defined in terms of fracture mechanics, material and laminate geometry parameters. Experimental data verified that this model can be used to predict the mean and variance associated with the growth of delamination at several load levels in a variable amplitude loading spectrum.

The location of the delamination in laminates was found to be load type specific. Under quasi-static loading the delamination formed in the interface with high singular opening edge stresses and a large mode I energy release rate component. Under fatigue loading the delamination formed in the interface with high singular opening edges stresses and a large mode II energy release rate component.

This stochastic model should provide more accurate estimates of structural reliability than wear out or damage tolerance models since the nature of the delamination growth phenomenon is inherent to the model. This model will allow statistically significant damage thresholds to be established for given variable amplitude service load spectrum.

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Ethics

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