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Modeling of Stress- Strain Curves of Drained Triaxial Test on Sand

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Abstract: This paper presents a hyperbolic mathematical model to predict the complete stress-strain curve of drained triaxial tests on uniform dense sand. The model was formed in one equation with many parameters. The main parameters that are needed to run the model are the confining pressure, angle of friction and the relative density. The other parameters, initial and final slopes of the stress strain curve, the reference stress and the curve-shape parameter are determined as functions of the confining pressure, angle of friction and the relative density using best fitting curve technique from the experimental tests results. Drained triaxial tests were run on clean white uniform sand to utilize and verify this model. These tests were carried out at four levels of confining pressure of 100, 200, 300 and 400 kPa. This model was used to predict the stress-strain curves for drained triaxial tests on quartz sand at different relative density using the data of Kouner[1]. The model predictions were compared with the experimental results and showed good agreements of the predicted results with the experimental results at all levels of applied confining pressures and relative densities.

Key words: Modeling, stress, strain, water, sand, triaxial

INTRODUCTION

Mathematical modeling of stress-strain curves of soil behavior obtained from triaxial tests gained great interest during the last three decades due to the need for calibration of the recent constitutive soil models that used in the analysis and prediction of the behavior of complex soil structures and soil/structure interaction problems using finite elements or finite difference method. An overview of constitutive models for soils was given by Lade[2]. The Hyperbolic Mathematical model is one of the simpler models that can simulate the nonlinear stress-strain curve of soil. Kondner[3] proposed a functional form based on hyperbolic stress-strain function which developed later by Duncan and Chang[4] and extended by Kulhawy *et al.*[5], but this model is limited to the hardening part of the stress-strain relationship.

A versatile model presented by Richard and Abbott[6] has been used to represent the stress-strain spectrum of different types of concrete. This model was used by Almusallam and Alsayed[7] to capture the complete stress-strain curves (hardening and softening parts) for normal, high strength and light weight concrete tested under various loading conditions. The authors have tried different empirical models to predict the complete stress-strain curve and concluded that some models need to have two different formulas to generate the complete stress-strain curve and other models could not take into account the influences of different factors influencing the stress-strain curves. Also, it was found that some models require complicated computations to evaluate their parameters. However, the model presented by Almusallam and Alsayed[7] was found to have the ability to consider the influence of different parameters affecting the stress-strain curve characteristics with only single parameter, which is the ultimate compressive strength of the concrete, needed to run the model. The other parameters of the model are determined using the best-fitting curve technique as a function of the compressive strength of the concrete. The model was presented by Almusallam and Alsayed[7] and found to provide good predictions of the experimental results for hardening and softening parts.

Alshenawy[8] extended the application of the above model to predict the complete differential cavity pressure-cavity volume change curve of an expansion of a thick-walled hollow cylinder test for both coarse and fine Ottawa sands. Based on t the experimental results, the confining pressure was chosen to be the input parameter to run the model. The other parameters were expressed as a function of the confining pressure and determined using the best-fitting curve technique. The computational results of the model were then compared with the experimental results and showed good agreements for both coarse and fine sands at all levels of confining pressures. The model was found to be able to predict the complete differential cavity pressure-cavity volume change curve at any value of confining pressure other than used in the tests.

The main objective of this paper is to utilize this model to predict the stress-strain curve of consolidated drained triaxial test on granular soil. The main

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parameters involved are the confining pressure, the angle of friction and the relative density. The other parameters were evaluated as a function of these parameters using best-fitting curve technique. The results of the suggested model was compared with the experimental results and found to be in a good agreement with the hardening and softening parts of the curve.

Proposed model of stress-strain curve: Basically, the suggested model consists of one equation which can be written for the triaxial test on soil to relate the change in deviator vertical stress (σ_d) to the vertical strain (ε_v) in the following form:

$$\sigma_d = \frac{\left(K - K_p\right)\ \varepsilon_v}{\left[1 + \left[\frac{\left(K - K_p\right)\ \varepsilon_v}{f_o}\right]^n\right]^{1/n}} + K_p\ \varepsilon_v \qquad (1)$$

As shown in Fig. 1, the parameters K, K_p are the initial and final slopes of the stress strain curve respectively, the parameter f_o is a reference stress and n is a curve-shape parameter given as:

$$n = -\frac{\ln\ m}{\ln\ \left[\frac{f_1}{f_o} - \frac{K_p}{K - K_p}\right]} \qquad (2)$$

where m is a constant and f_1 can be given as:

$$f_1 = \sigma_d^p \left[2\ \frac{\varepsilon_1}{\varepsilon_v^p} - \left[\frac{\varepsilon_1}{\varepsilon_v^p}\right]^2\right] \qquad (3)$$

where σ_d^p is the peak deviator vertical stress, ε_v^p is the corresponding vertical strain and ε_1 can be written as:

$$\varepsilon_1 = \frac{f_o}{K - K_p} \qquad (4)$$

The advantage of this model is its ability to predict the complete stress-strain curve for both hardening and softening parts.

Experimentation: A set of consolidated drained triaxial tests was carried out in this study to calibrate the proposed model. The tests were run on local white uniform sand and the grain size distribution curve is shown in Fig. 2. The sand grains are almost rounded with an average grain size diameter of about 0.5 mm. The major properties of this sand are presented in Table 1. According to the Unified Classification System, this soil is poorly graded sand (SP).

The experiments were performed using triaxial testing apparatus using consolidated drained condition. The triaxial specimen was prepared at a high relative density of 95% using split mold with 35.5 mm in diameter and 71 mm in height. The value of the pore water pressure parameter (B) was not less than 97% for good degree of saturation. In the experimental program, the specimens were tested at four levels of confining

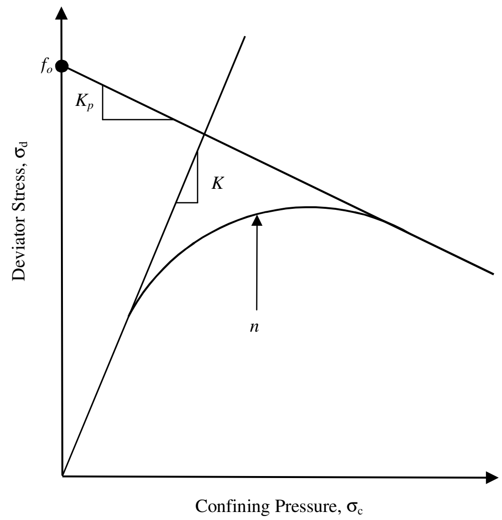

Fig. 1: Illustration of the model's parameters

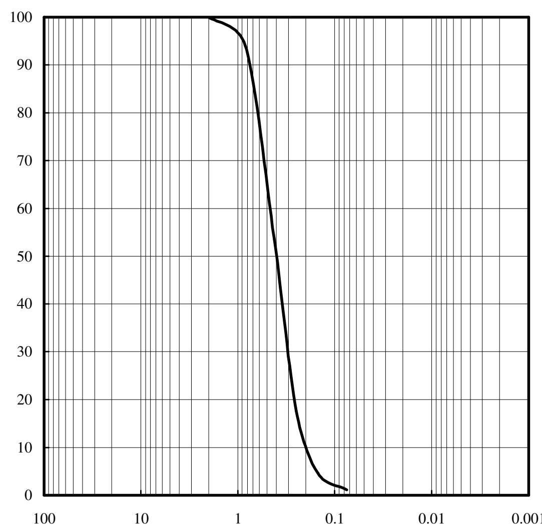

Fig. 2: Gradation curve of the tested sand

pressure of 100, 200, 300 and 400 kPa at a constant rate of displacement of 0.3 cm sec^{-1}.

Evaluation of the model parameters: The parameters that needed to be determined in order to execute the model in Eq. (1) are: K, K_p, f_o, ε_v^p, σ_d^p, m and n. These parameters were obtained based on the experimental results as functions of the main parameters which include the confining pressure, the angle of friction and the relative density using the best-fitting curve technique.

i. The parameter K: The initial slope (K) of the experimental stress-strain curves, which represent the initial tangent modulus of elasticity of the sand, could be expressed as a function of the confining pressure (σ_c) by a linear equation (Fig. 3) as:

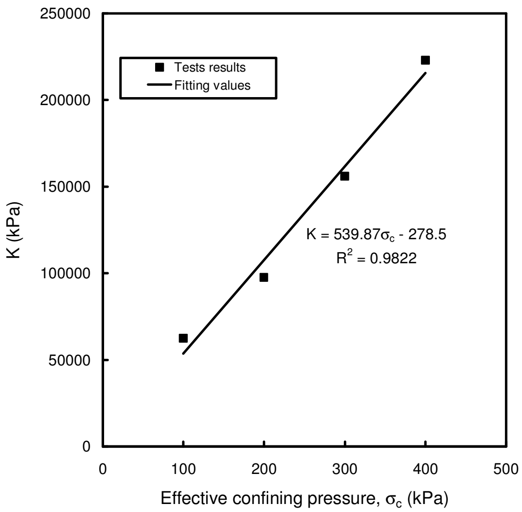

Fig. 3: Variation of K with confining pressure at D_r=95%

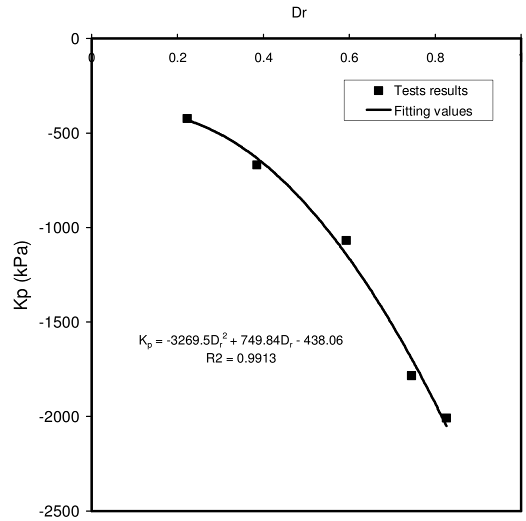

Fig. 6: Variation of K_p with relative density at σ_c=207 kPa

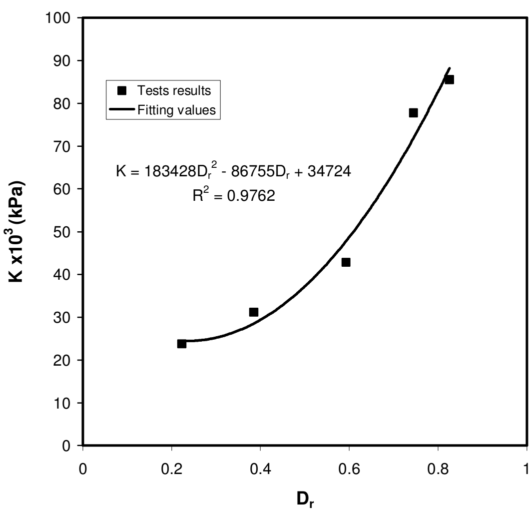

Fig. 4: Variation of K with relative density at σ_c=207 kPa

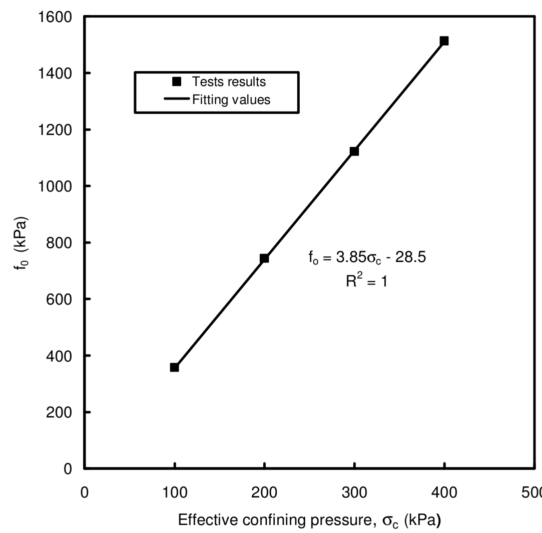

Fig. 7: Variation of f_o with confining pressure at D_r =95%

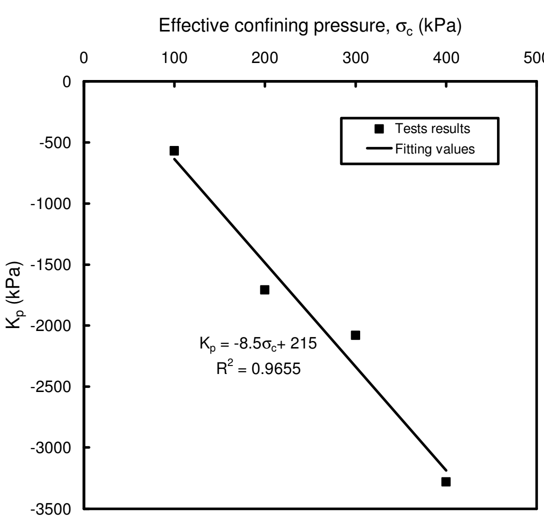

Fig. 5: Variation of K_p with confining pressure at D_r=95%

$$K = 540\sigma_c - 278.5 \quad (5)$$

Figure 4 illustrates the variation of K against the relative density (D_r) using the data of Kouner[1] where K expressed as:

$$K = 183428D_r^{\ 2} - 86755D_r + 34724 \quad (6)$$

Where the units of K and σ_c are in kPa.

ii. The parameter K_p: The final slopes (K_p) of the experimental curves can be expressed as a function of the confining pressure by linear equations, as shown by Fig. 5. K_p can be expressed as:

$$K_p = -(8.5\sigma_c - 215) \quad (7)$$

or as a function of the relative density according to the data of Kouner[1] as shown by Fig. 6, as

$$K_p = -3269.5D_r^{\ 2} + 749.84D_r - 438.06 \quad (8)$$

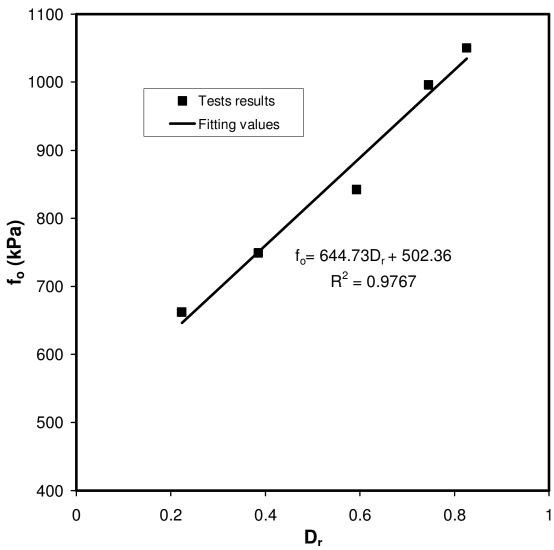

Fig. 8: Variation of f_o with relative density at σ_c=207 kPa

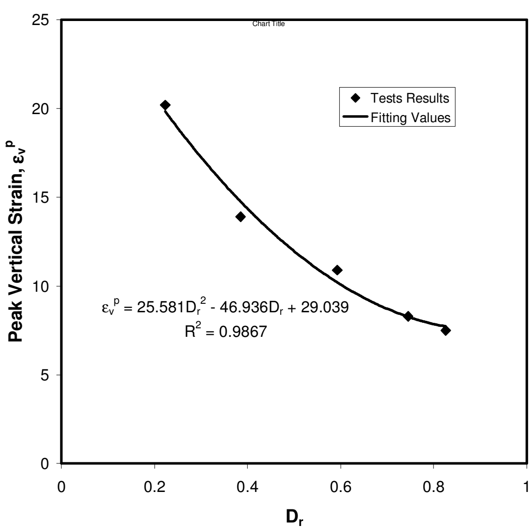

Fig. 9: Variation of peak vertical strain with relative density at σ_c=207 kPa

Where the units of K_p and σ_c are in kPa.

iii. The parameter f_o: Figure 7 shows a linear relationship between the reference stress (f_o) and the confining pressure which could be expressed as:

$$f_o = 3.85\sigma_c - 28.5 \quad (9)$$

or as a function of the relative density according to the data of Kouner[1] as shown in Fig. 8 as

$$f_o = 644.7D_r + 502 \quad (10)$$

Where the units of f_o and σ_c are in kPa.

iv. The parameter ε_v^p: According to the tests results of Kouner[1], ε_v^p was found to vary with the variation of the relative density as shown in Fig. 9 according to the following relationship

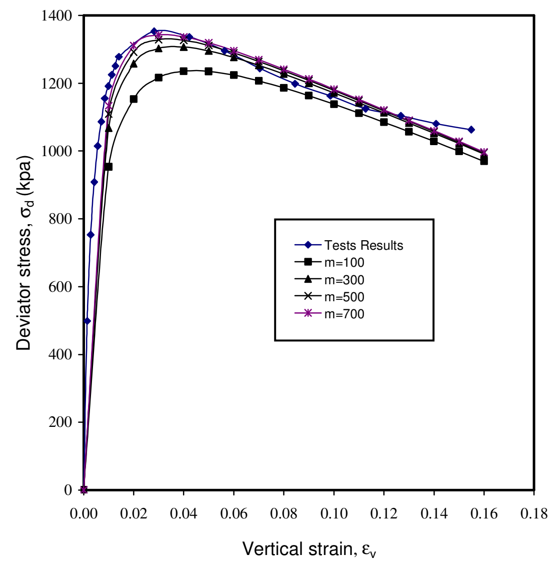

Fig. 10: Comparison between the predicted stress-strain curves at different values of m at D_r=95% and σ_c=400 kPa

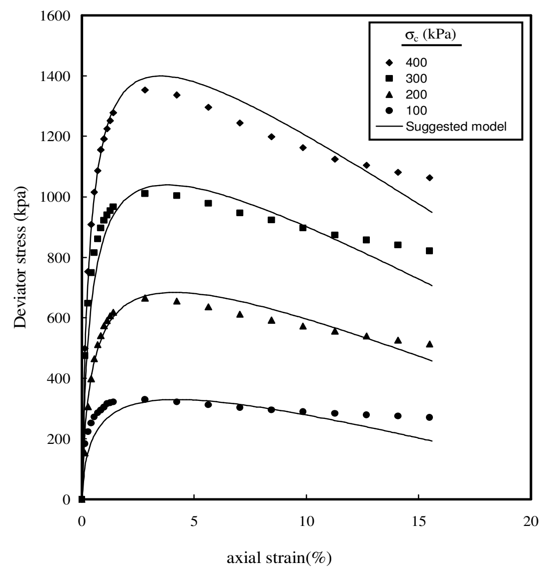

Fig. 11: Comparison between the predicted and the experimental results of the deviator stress at D_r=95%

$$\varepsilon_v^p = 25.58D_r^{\ 2} - 46.94D_r + 29.04 \quad (11)$$

This relationship was used in the calibration of the model parameter *m* using the results of tests that conducted in this study at a relative density of 95% and found to give good results.

v. The parameter σ_d^p: According to Mohr-Coulomb failure criteria, the relationship between the peak deviator stress σ_d^p and the confining pressure for sand (at C=0, where C is the soil cohesion) is given by

$$\sigma_d^p = \sigma_c\left[\frac{2\sin\phi}{1-\sin\phi}\right] \quad (12)$$

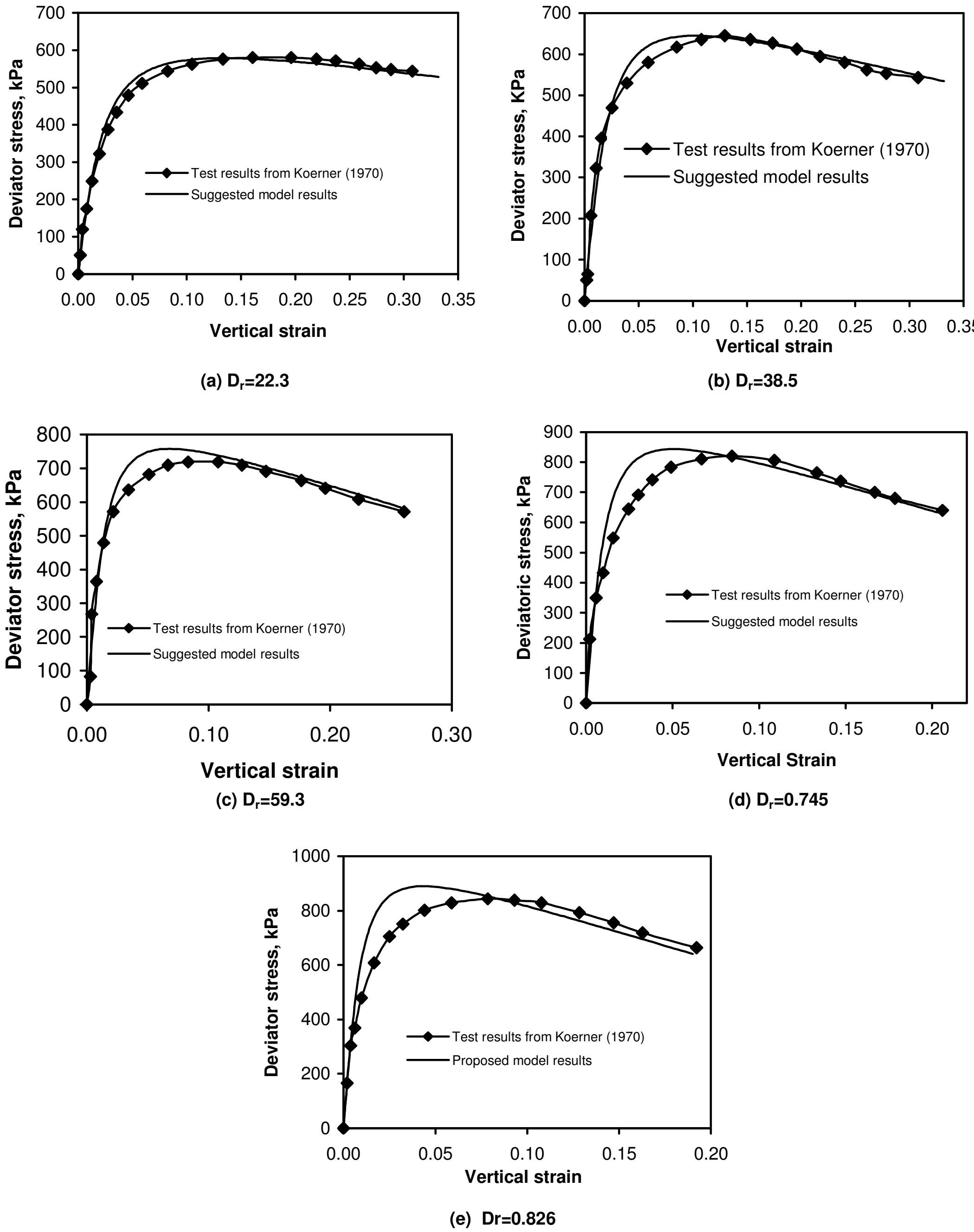

Fig. 12: Comparison between the predicted and the experimental results of the deviator stress at confining pressure of 207 kPa

Where ϕ is the angle of friction of the sand.

vi. The parameter *m*: The parameter m will be evaluated after the calibration of the model.

vii. The parameter n: The parameters n can be calculated from Eq. (2).

Calibration of the proposed model: The calibration of the model depends on the determination of the parameter m since the other parameters are evaluated directly from the tests results. This was carried out by testing the model for different values of m as shown by Fig. 10 using the tests results at confining pressure of 400 kPa, the best value was found to be when m=700.

It was found that the best value that is applicable to all the confining stress levels with marginal deviation from the test results when m=300 as shown by Fig. 11. Hence a value of m of m=300 was used to verify the model.

Verification of the proposed model: To verify the proposed model, it was used to predict the stress-strain curves for quartz sand tested using consolidated drained triaxial tests that were published by Kouner[1]. The prediction of the stress-strain curves at different relative densities via the proposed model is shown Fig. 12. This figure shows that the predicted curves by the suggested model are in a good agreement with the experimental curves in both hardening and softening parts for all levels of the relative density. In the case of dense sand or high confining pressure where the stress-strain curve has a well defined peak value and at high strain value of about 15%, the predicted deviator stress values become lower than those of the experimental results as the strain increases where the experimental values stay almost constant as the strain increases. However, in most of the practice problems, we may not need to go beyond a vertical strain of 15%. This concludes that this simple model is efficient in predicting the stress-strain curve of the sandy soil at any value of confining pressure and relative density using the consolidated drained triaxial test.

CONCLUSION

In this paper, a simple hyperbolic mathematical model is proposed to generate the complete deviator vertical stress vs. vertical strain curve of the consolidated drained triaxial test on sand. The model has the advantage of considering the influence of different factors affecting the stress-strain curve characteristics including the confining pressure, angle of friction and relative density. The model was calibrated and verified using two sets of data of consolidated drained triaxial tests at different levels of confining pressures and relative density. The first set of data consists of four tests on dense sandy samples that were tested at relative density of 95% and at four levels of confining pressure of 100, 200, 300 and 400 kPa. The other set consists of five tests that were published by Kouner[1] at confining pressure of 207 kPa and different relative densities of 22.3, 38.5, 59.3, 74.5 and 82.6. The three main parameters that were used to run the model include the confining pressure, the angle of friction and the relative density. The other parameters of the model were determined using the best fitting curve technique as function of these main parameters. The model prediction curves were compared with the experimental ones and found to provide good agreements at all the levels of confining pressure and relative density for the hardening and softening parts of the stress-strain curve.

List of Symbols

D_r = relative density
E = void ratio
f_0 = the reference stress
f_1 = constant
Gs = specific gravity
K = the initial slope of the stress strain curve
K_p = the final slope of the stress strain curve
n = a curve-shape parameter
m = constant
ϕ = angle of friction
γ_d = dry unit weight
σ_d = deviator vertical stress
σ_d^p = the peak deviator vertical stress
ε_v = the vertical strain
ε_v^p = the peak vertical strain
ε_1 = constant

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