American Journal of Applied Sciences 2 (1): 436-439, 2005
ISSN 1546-9239

Design of Flywheel with a Moving Hole

Jihad Said Addasi
Department of Basic Sciences, Tafila Applied University College
Al–Balqa' Applied University, P.O. Box 40, Al-Eys-66141, Tafila, Jordan

Abstract: A special apparatus has been designed to study linear, circular, and rotational motions. This apparatus consists of a flywheel, rotating in horizontal plane by the mean of a hanging mass. This flywheel carries a hole, which can be moved on a radial rail. This hole can be fixed at desired distance from the flywheel axis. Putting a metallic sphere on this hole gives centripetal motion with certain radius to this sphere. The moving hole acts by normal force on the metallic sphere and moves it in circular path. The increasing angular velocity of the metallic sphere has its maximum value when the sphere leaves the hole. Theoretical and experimental analysis for this apparatus shows that the centripetal acceleration depends only on the geometry of both sphere and hole. The designed apparatus has a very simple design and does not have any undesired forces.

Key words: Design, Flywheel, Moving Hole, Circular Motion

INTRODUCTION

It is difficult to study linear, circular and rotational motions at the same time using one apparatus [1, 2]. However, it is easy to study linear and rotational motions using a flywheel, while a flywheel can not be used to study a circular motion. A simple apparatus shown in Fig. 1a can be used to study a circular motion. A string (2) passing through a thin tube (1) is connected with rotating (m) and hanging (M) masses. By doing some vibration for thin tube, the mass m moves in circular path and tenses the string with tension force $\vec{T}$. The circular motion of mass m will be established when the radius r of circular path is constant and the mass M hangs at a certain height. This apparatus has the following disadvantages: 1. It is difficult to stabilize a uniform circular motion by using this apparatus, 2. The friction force between a string and a thin tube can not be neglected, 3. there is no explicit relation between linear and circular motions, because there is no linear motion along straight line. These difficulties may be avoid by using a flywheel with a moving hole, which means that the suggested apparatus can be used to study linear, rotational and circular motions at the same time.

Design of the Suggested Apparatus: The hanging mass M under the action of its weight $\vec{W} = M\vec{g}$ and tension force $\vec{T}$ in a thin string (4) moves downward along a straight line (linear motion) with constant linear acceleration $\vec{a}_M$. The string wrapped around a ring (5) with radius R. This ring has a common rotational axis (7) with a flywheel (1). The string (4) acts on the flywheel (1) by torque with magnitude τ=RT. Having constant values of the hanging mass M, the radius R and moment of inertia I_{tot} of flywheel with a metallic sphere, then the torque $\vec{\tau}$, angular acceleration of flywheel $\vec{\alpha}$ and linear acceleration of hanging mass $\vec{a}_M$ will be constant. The moment of inertia I_{tot} is constant, because $I_{tot} = I_{wh} + I_m$; $I_m << I_{wh}$, where, I_{wh} , I_m – moment of inertia of flywheel and of the metallic sphere of mass m, respectively, $I_m = mr^2 < 0.005$ kg·m^2, m < 0.06 kg, r < 0.3 m, $I_{wh} \approx 0.04$ kg·m^2.

The radius R of ring can be used to find the following relations between rotational motion of the flywheel and linear motion of hanging mass M:

$$S_M=R\theta;\ V_M=R\omega;\ a_M=R\alpha \qquad (1)$$

Where, S_M, V_M and a_M are the magnitude of linear: displacement; velocity and acceleration for hanging mass (M), θ, ω, α are the magnitude of angular: displacement; velocity and acceleration for flywheel.

A metallic sphere (m) was put on the moving hole (3) with cylindrical form of radius (b) with vertical central axis. In addition the moving hole can be moved on a radial rail (2), and can be fixed at certain distance (r) from rotational axis (7). When the flywheel rotates, the metallic sphere moves in circular path (8) of radius (r). The moving hole acts on the metallic sphere (m) with a normal force $\vec{N}$. This normal force has an increasing magnitude and a changing direction with time. The normal force has its maximum value $\vec{N}^{max}$ when the metallic sphere leaves the moving hole. The normal force $\vec{N}^{max}$ is directed to the center of the metallic sphere and makes the angle β with vertical direction (Fig. 1b). For this angle the following equation can be written:

$$\text{Sin}(\beta)=b/a \qquad (2)$$

where, b < a, b- the radius of the metallic sphere,

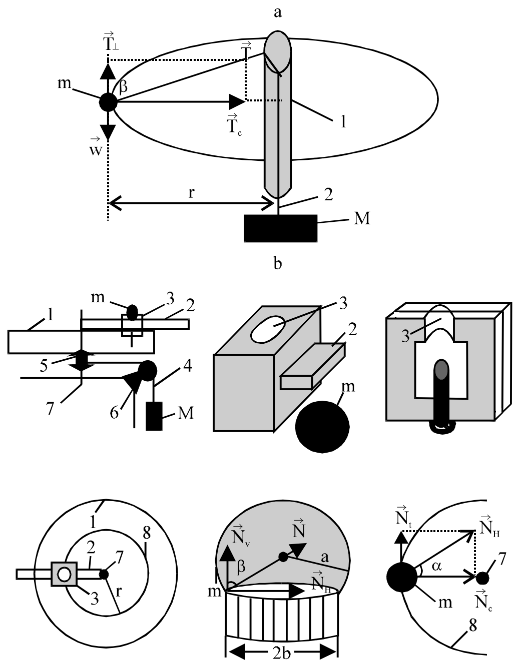

Fig. 1: a) Construction of a Simple Apparatus for Circular Motion. Where, 1-a Thin Tube, 2-A String, r-Radius of Circular Path, m, M-Rotating, Hanging Masses, $\vec{w}$-Weight of Rotating Mass, $\vec{T}$-Tension Force, β-Angle between $\vec{T}$ and Vertical Direction
b) Construction of the Suggested Apparatus. Where: 1-A Flywheel, 2-A Rail; 3-A Moving Hole, m-A Metallic Sphere, 4-A Thin String, 5-A Ring, 6-A Pulley, M-A Hanging Mass, 7-Rotational Axis, 8-Circular Path, r-Radius of Circular Path, a-Radius of the Metallic Sphere, b-Radius of the Moving Hole, β-Angle between Normal Force $\vec{N}$ and Vertical Direction

a- the radius of the moving hole.
The metallic sphere leaves the moving hole with acceleration:

$$\vec{a}_m^{max} = \vec{a}_c^{max} + \vec{a}_t^{max} \tag{3}$$

where, $\vec{a}_m^{max}$ makes the angle γ with centripetal of circular motion; $\vec{a}_c^{max}$ - the centripetal acceleration and $\vec{a}_t^{max}$ - the tangential acceleration of the metallic sphere m.
When the metallic sphere leaves the moving hole, the following equations can be written:

$$\vec{N}^{max} = \vec{N}_V^{max} + \vec{N}_H^{max} = \vec{N}_V^{max} + \vec{N}_c^{max} + \vec{N}_t^{max},$$
$$N_V^{max} = N^{max}\cos(\beta) = mg;$$
$$N_H^{max} = mg\tan(\beta) = ma_m^{max} = mgb/\sqrt{a^2 - b^2};$$
$$N_c^{max} = mr(\omega^{max})^2 = N^{max}\sin(\beta)\cos(\gamma);$$
$$N_t^{max} = mr\alpha^{max} = N^{max}\sin(\beta)\sin(\gamma);$$
$$\vec{a}_m^{max} = \vec{a}_c^{max} + \vec{a}_t^{max};\ a_m$$
$$= (r/R)(g\frac{MR^2}{MR^2 + I_{wh}})\sqrt{1 + \frac{4s^2}{R^2}} = g\frac{b/a}{\sqrt{1-(b/a)^2}} \tag{4}$$

Where, $\vec{N}_V^{max}, \vec{N}_H^{max}, \vec{N}_c^{max}$ and $\vec{N}_t^{max}$-the components of normal force when the metallic sphere leaves the moving hole: along vertical direction, in horizontal plane, directed to center of circular motion and directed tangentially to the circular path, g-acceleration due to gravity, ω^{max}, α^{max}- the angular: velocity, acceleration of the metallic sphere when it leaves the moving hole, s-the displacement of mass M until the sphere leaves the hole.
At the same time the hanging mass M moves with constant acceleration of magnitude:

$$a_M = g\frac{MR^2}{MR^2 + I_{tot}} \tag{5}$$

For not big hanging mass M < 0.5 kg; $a_M < 0.07\ m/s^2$ the metallic sphere moves uniformly in circular path. In this case the magnitude of tangential acceleration is enough smaller than the magnitude of centripetal acceleration ($a_t^{max}/a_c^{max} < 0.05$) of this metallic sphere. The equations (4) can be written for not big hanging mass in the form:

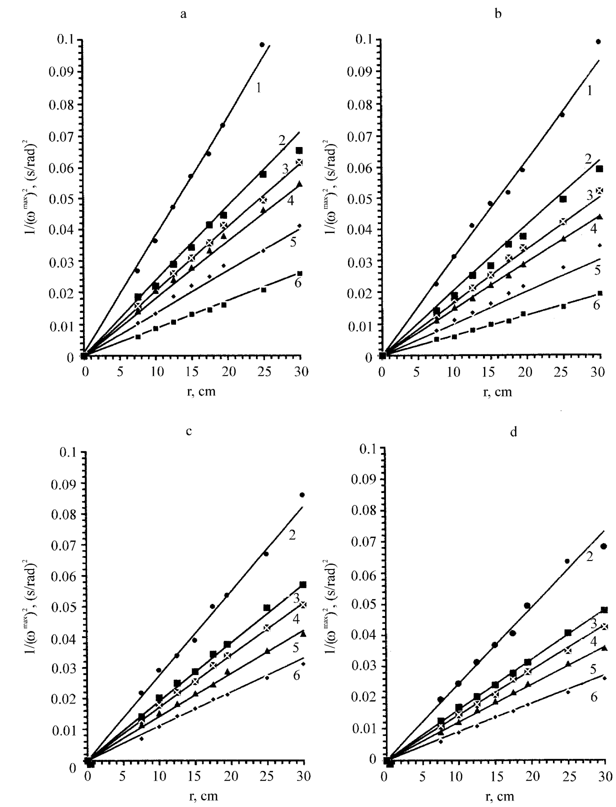

Fig. 2: The Inverse of the Squared Angular Velocity ($1/(\omega^{max})^2$, $(s/rad)^2$) Versus Radius r, cm of Circular Path for Radius of Moving Hole: (a) 3.65 mm, (b) 4.10 mm, (c) 4.50 mm and (d) 4.80 mm. Where, Curves: 1, 2, 3, 4, 5 and 6 are taken for Metallic Sphere with Radius: 4.75 mm, 6.25 mm, 8.00 mm, 8.75 mm, 9.50 mm and 12.50 mm, respectively

$$\vec{N}^{max} = \vec{N}_V^{max} + \vec{N}_c^{max};$$
$$N_V^{max} = N^{max}\cos(\beta) = mg;$$
$$N_c^{max} = mg\tan(\beta) = ma_c^{max} = mg\, b\big/\sqrt{a^2-b^2};$$
$$N_c^{max} = mr(\omega^{max})^2 = N^{max}\sin(\beta);$$
$$\bar{a}_m^{max} = \bar{a}_c^{max};\ a_c$$
$$= (2sr/R^2)\left(g\frac{MR^2}{MR^2+I_{wh}}\right) = g\frac{b/a}{\sqrt{1-(b/a)^2}} \qquad (6)$$

RESULTS AND DISCUSSION

For experimental data the motion of flywheel begins from rest (initial angular velocity is zero, $\omega_0=0$), and the radius of ring is taken R=2.36 cm. Each of the used metallic spheres has a mass less than 0.06 kg and with radius 4.75<a<12.5mm. The hanging mass M is not more than 0.5 kg. Figure 2 shows the dependence of inverse of squared angular velocity ($1/(\omega^{max})^2$) for the metallic sphere on the radius (r) of a circular path.

Figure 2a-d corresponds with hole of radius: 3.65mm (a), 4.10 mm (b), 4.50 mm (c) and 4.80 mm (d). The

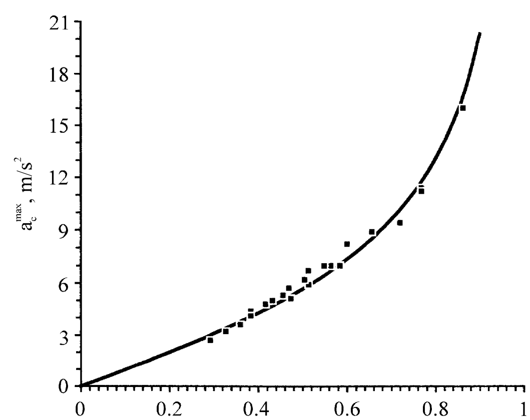

Fig. 3: Dependence of Centripetal Acceleration (a_c^{max}) on the Ratio of Radius of the Moving Hole to the Radius of the Metallic Sphere (b/a): for Experimental Results (points) and Theoretical Calculations (Solid Line)

metallic spheres with radii: 4.75, 6.25, 8.00, 8.75, 9.5 and 12.50 mm are used for getting curves: 1, 2, 3, 4, 5 and 6, ` respectively. The magnitude of centripetal acceleration a_c^{max} and the ratio of spheres radius to radius of moving hole (b/a) are determined for each linear dependence in Fig. 2. These experimental results are illustrated in Fig. 3 by points. Using equations (6), the magnitude of centripetal acceleration a_c^{max} of the metallic sphere is calculated theoretically as a function of the ratio (b/a). The theoretical dependence of centripetal acceleration a_c^{max} of the metallic sphere on the ratio (b/a) is illustrated in Fig. 3 by solid line, which agrees with experimental results (experimental points). Equation $a_c = g\frac{b/a}{\sqrt{1-(b/a)^2}}$ and Fig. 3 show that, the centripetal acceleration (a_c^{max}) depends only on the geometry of both of the moving hole and the metallic sphere. While the centripetal acceleration (a_c^{max}) is independent of the radius of circular path (r). It means, that the centripetal force ($\vec{N}_c^{max}$) is constant for the same moving hole and the same metallic sphere for any radius (r) of centripetal path.

CONCLUSION

The study of linear, circular and rotational motions at the same time is available due to the moving hole, carrying on the flywheel. The centripetal acceleration of metallic sphere, when it leaves the moving hole, depends only on the geometry of both of the moving hole and the metallic sphere. The designed apparatus doesn't have any undesired forces and has economical and simple design.

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